JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY: KAKINADA
KAKINADA – 533 003, Andhra Pradesh, India
DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

III Year - II Semester		L	T	P	C
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WIRED and WIRELESS TRANSMISSION DEVICES

Course objectives:
The student will be able to
- understand the applications of the electromagnetic waves in freespace.
- introduce the working principles of various types ofantennas
- discuss the major applications of antennas with an emphasis on how antennas are employed to meet electronic systemrequirements.
- understand the concepts of radio wave propagation in theatmosphere.

UNIT I
MICROWAVE TRANSMISSION LINES: Introduction, Microwave Spectrum and Bands, Applicationsof Microwaves. Rectangular Waveguides – TE/TM mode analysis, Expressions for Fields, Characteristic Equation and Cut-off Frequencies, Filter Characteristics, Dominant and Degenerate Modes, Sketches of TE and TM mode fields in the cross-section, Mode Characteristics – Phase and Group Velocities, Wavelengths and Impedance Relations; Power Transmission and Power Losses in Rectangular Guide, Impossibility of TEM mode.Related Problems, Excitation techniques-waveguides
MICROSTRIP LINES– Introduction, Z_0 Relations, Effective Dielectric Constant, Losses, Q factor

UNIT II
ANTENNA FUNDAMENTALS: Introduction, Radiation Mechanism – single wire, 2 wire, dipoles, Current Distribution on a thin wire antenna. Antenna Parameters - Radiation Patterns, Patterns in Principal Planes, Main Lobe and Side Lobes, Beam widths, Polarization, Radiation Intensity, Directivity, Gain Antenna Apertures, Aperture Efficiency, Effective Height, illustrated Problems.

UNIT III
THIN LINEAR WIRE ANTENNAS: Retarded Potentials, Radiation from Small Electric Dipole, Quarter wave Monopole and Half wave Dipole – Current Distributions, Evaluation of Field Components, Power Radiated, Radiation Resistance, Beam widths, Directivity, Effective Area and Effective Height, Antenna Theorems – Applicability and Proofs for equivalence of directional characteristics, Loop Antennas: Small Loops - Field Components, Concept of short magnetic dipole, D and R_r relations for smallloops.
ANTENNA ARRAYS: Principle of Pattern Multiplication, N element Uniform Linear Arrays – Broadside, End-fire Arrays, Binomial Arrays, Arrays with Parasitic Elements. Yagi-Uda Arrays, Folded Dipoles and theircharacteristics.

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UNIT IV
NON-RESONANT RADIATORS: Introduction, Traveling wave radiators, Long wire antennas, Rectangular Patch Antennas –Geometry and Parameters, Impact of different parameters on characteristics. Helical Antennas, Geometry, basic properties
VHF, UHF AND MICROWAVE ANTENNAS: Reflector Antennas: Corner Reflectors. Parabolic Reflectors – Geometry, characteristics, types of feeds, F/D Ratio, Spill Over, Back Lobes, Aperture Blocking, Cassegrain Feeds.
Horn Antennas – Types, Optimum Horns, Lens Antennas – Geometry, Features, Dielectric Lenses and Zoning, Applications.

UNIT V
WAVE PROPAGATION: Concepts of Propagation – frequency ranges and types of propagations. Ground Wave Propagation–Characteristics, Fundamental Equation for Free-Space Propagation, Basic Transmission Loss Calculations, Space Wave Propagation–Mechanism, LOS and Radio Horizon, Tropospheric Wave Propagation – Radius of Curvature of path, Effective Earth's Radius, Effect of Earth's Curvature, Field Strength Calculations.
ANTENNA MEASUREMENTS – Patterns, Set Up, Distance Criterion, Directivity, VSWR, Impedance and Gain Measurements (Comparison, Absolute and 3-Antenna Methods)

TEXT BOOKS

1. Electromagnetic Waves and Radiating Systems – E.C. Jordan and K.G. Balmain, PHI, 2nd Edition,2000.
2. Antennas and wave propagation- Sisir K Das, Annapurna Das, TMH,2013.

REFERENCES

1. Antennas – John D. Kraus, McGraw-Hill, 2nd Edition,1988.
2. Transmission and Propagation – E.V.D. Glazier and H.R.L. Lamont, The Services Text Book of Radio, vol. 5, Standard Publishers Distributors,Delhi,2009.
3. Antennas and wave propagation by Prof G S N Raju, Pearsion Publications, First impression,2016

Course Outcomes:
After going through this course the student will be able to

- Identify basic antennaparameters.
- Design and analyze wire antennas, loop antennas, reflector antennas, lens antennas, horn antennas and micro stripantennas
- Quantify the fields radiated by various types ofantennas
- Design and analyze antennaarrays
- Analyze antenna measurements to assess antenna'sperformance
- Identify the characteristics of radio wavepropagation

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MICROWAVE TRANSMISSION LINES

Introduction :—

Microwave frequencies lie in the range of 1GHz to 1000GHz

Main advantage is antenna size is reduced.

Frequency	Band designations
3 - 30 Hz	ultra low freq's (ULF)
30 - 300Hz	Extra low freq's (ELF)
300 - 3KHz	voice frequencies
3K - 30KHz	very low freq (VLF)
30K - 300KHz	Low freq (LF)
300K - 3MHz	Medium freq
3M - 30MHz	High freq
30M - 300 MHz	very high freq (VHF)
300M - 3GHz	ultra high freq (UHF)
3G - 30GHz	Super high freq (SHF)
30G - 300GHz	Extreme high freq (EHF)
300G - 3THz	
3T - 30THz	Infrared freq
30T - 300THz	

Micro wave freq's

***Advantages of Microwaves :-

1. Increased bandwidth availability
2. Increased directivity $\qquad G = D = \frac{4\pi A_e}{\lambda^2}$
3. Fading Effect & Reliability
4. Tx & Rx power (↓↓ (mw))
5. Transparency property of microwaves (300 MHz - 10 GHz)

[Fading - Fluctuation in signal strengths]

① Increased bandwidth availability :-

Microwave freq has large bandwidth when compared to short waves, medium waves and ultra waves

Microwave freq's consist of 1000 sections of freq. bands and any one of these 1000 sections may be used to transmit all radio, TV signals and other Communication signals

② Improved directivity :-

At microwave freq's directivity is increased and beam width ~~bandwidth~~ is decreased ($\because \theta \propto \lambda/D$)

For parabolic reflector antenna. directivity

$$D = \frac{4\pi A_e}{\lambda^2}$$

At microwave freq's λ is decreased & D is increased

For parabolic reflector antenna $B = 140/(D/\lambda)$

At 30 GHz ($\lambda = 1$ cm) for 1° beam width $D = 140$ cm

At 30 MHz ($\lambda = 100$ cm) for 1° beam width $D = 140$ m

Beam width $BW = \frac{140\lambda}{d}$

where D is diameter of the reflector At microwave freq's Antenna size is very small

③ Fading Effect and reliability :-

Fading Effect due to variation in transmission media is more Effective at lower freq's. Due to line of sight (LOS) propagation at higher freq's there is less fading Effect ~~and~~ hence microwave communication is more reliable.

④ power Requirements :-

Tx/Rx power requirements are very low at microwave freq's compared to that of short waves

⑤ Transparency property of microwaves :-

Microwave freq band ranging from 300 MHz – 10 GHz. are freely propagat through the ionized layers surrounding the earth. The presence of such a transparent window in microwave region facilitates the study of microwave radiations from Sun and Stars.

Applications :-

Microwave frequencies have band range of applications in modern technology. Most important among them are in along distance communication, RADAR's, radio astronomy etc.

① Telecommunications :—

International telephones and TV, space Communication, telemetry Communication link for railways etc

② RADAR's (Radio detection and ranging)

These are used to detect aircrafts, track and guide super sonic missiles, observe weather Conditions, air traffic Control (ATC) police speed detectors etc

3. Commercial and Industrial Applications use heat property of microwaves.

Microwave oven, Drying machine, food processing industry, rubber industry, mining ores, dry links and biomedical applications

4. Electronic warfare ECM | ECCM system, spread spectrum system.

ECM - Electronic Counter Measurement

ECCM - Electronic Counter Counter measurement

Types of EM wave

EM wave

TE TM TEM

1. Transverse electric (TE) wave :—

In TE wave, The Component of electric field vector lies in a plane transverse (or) perpendicular to the direction of propagation. where as Component of magnetic field vector lies in the direction of propogation. In TE waves $E_z = 0$, $H_z \neq 0$, if the wave is propagating in Z-direction.

② Transverse magnetic (TM) wave :-

In TM wave the component of magnetic field vector lies in a plane transverse or $\perp^{lar}$ to the direction of propagation where as the component of Electric field vector lies in the direction of propogation. In TM wave $H_z = 0$, $E_z \neq 0$

③ Transverse electromagnetic (TEM) wave :-

In TEM wave, both Electric and magnetic field vectors lies in a plane transverse or $\perp^{ar}$ to the direction of propagation ($E_z = H_z = 0$)

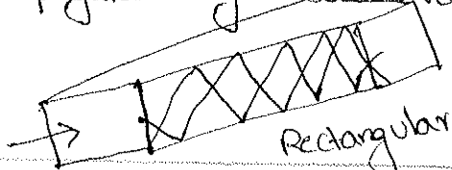
Not :- In waveguides, TEM wave does not Exist

WAVEGUIDES :-

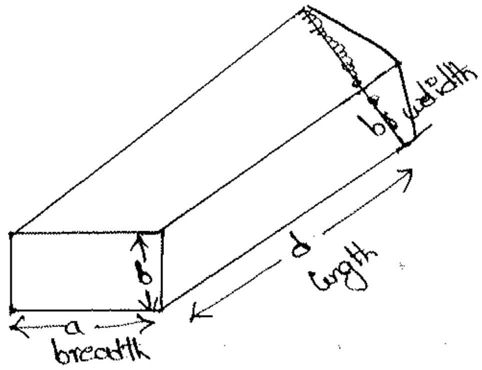
If freq is greater than 3GHz, transmission of that electromagnetic wave along Tx lines and coaxial cables is very difficult due to radiation losses and dielectric losses

A hollow metalic tube is be used to transmit EM waves at higher freq's and that tube is called waveguide

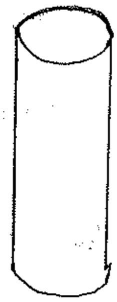
In waveguide the wave is propogated by successive reflections from inner walls of waveguide


Rectangular waveguide

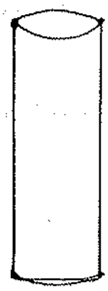
Types of waveguides :—



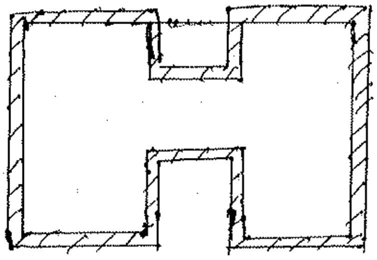
Rectangular waveguide



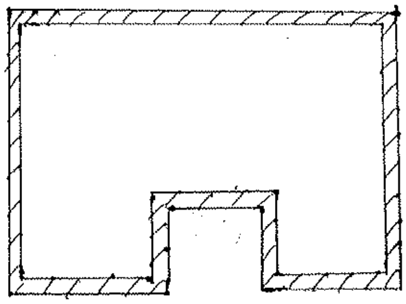
Circular wave guide



Elliptical waveguide



Double ridged wave guide



Single riged

Always a>> b

-Any shape of Crosssection of waveguide can support EM wave but irregular shapes are very difficult to analyse. In general rectangular and Circular waveguides are most popularly used.

Analysis of TE and TM waves in rectangular waveguides :-

TE waves :-

$$E_z = 0$$

$$H_z = A \cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \; e^{-j\beta z}$$

$$E_x = \frac{-j\omega\mu}{k^2}\frac{\partial H_z}{\partial y} \Rightarrow E_x = \frac{j\omega\mu}{k^2} A\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

$$E_y = \frac{j\omega\mu}{k^2}\frac{\partial H_z}{\partial x} \Rightarrow E_y = \frac{-j\omega\mu}{k^2} A\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

$$H_x = \frac{-j\beta}{k^2}\frac{\partial H_z}{\partial x} \Rightarrow H_x = \frac{j\beta}{k^2} A\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

$$H_y = \frac{-j\beta}{k^2}\frac{\partial H_z}{\partial y} \Rightarrow H_y = \frac{j\beta}{k^2} A\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

where a & b are dimensions of waveguide

m & n are mode no's. wave is designated as TE_{mn} (or) TM_{mn}

β is phase constant

(i) if $m=0$ & $n=0$, then

$E_x = 0 \quad H_x = 0$

$E_y = 0 \quad H_y = 0$

TE_{00} mode ~~has~~ does not Exist

(ii) If $m=1$, $n=0$, then $E_x = 0$, $H_y = 0$

E_y & $H_x \neq 0$

TE_{10} mode Exists

(iii) If $m=0$, $n=1$, then E_x & $H_y \neq 0$

E_y & $H_x = 0$

TE_{01} mode Exists

TM waves :—

$$H_z = 0$$

$$E_z = A \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

$$E_x = \frac{-j\beta}{k^2}\frac{\partial E_z}{\partial x} \Rightarrow E_x = \frac{-j\beta}{k^2} A\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

$$E_y = \frac{-j\beta}{k^2}\frac{\partial E_z}{\partial y} \Rightarrow E_y = \frac{-j\beta}{k^2} A\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right) e^{-j\beta z}$$

$$H_x = \frac{j\omega\epsilon}{k^2}\frac{\partial E_z}{\partial y} \Rightarrow H_x = \frac{j\omega\epsilon}{k^2} A\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

$$H_y = \frac{-j\omega\epsilon}{k^2}\frac{\partial E_z}{\partial x} \Rightarrow H_y = \frac{-j\omega\epsilon}{k^2} A\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

(i) If $m=0$, $n=0$ then

$H_z = E_z = 0$; $E_x = E_y = H_x = H_y = 0$

TM_{00} does not Exists

(ii) If $m=1$, $n=0$ then

$E_x = 0$, $E_y = 0$, $H_x = 0$, $H_y = 0$

TM_{10} does not Exists

(iii) If $m=0$, $n=1$ then

$E_x = 0$, $E_y = 0$, $H_x = 0$, $H_y = 0$

TM_{01} does not Exists

(iv) If $m=1$, $n=1$ then

$E_x = E_y = H_x = H_y \neq 0$

TM_{11} Exists and it is the starting mode.

Expression for cut off freq in rectangular waveguides

$$k^2 = p^2 + \omega^2\mu\varepsilon$$

$$= \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

In waveguides

$$k^2 = p^2 + \omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

It is characteristic equation.

$$p = \alpha + j\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

where p is propagation constant $= \alpha + j\beta$

α = attenuation const

β = phase const

$\omega = 2\pi f$

m, n = mode no's

a, b = dimensions of rect waveguide

At lower freq's $\omega^2\mu\varepsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

The propagation const becomes +ve & real and is equal to attenuation const. i.e; wave is attenuated

At higher freq's $\omega^2\mu\varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$, the propagation const becomes imaginary and is equal to phase const i.e wave is propagated

At some freq, $\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$ and propagation const is zero and that freq is called cut off frequency (or) threshold frequency.

At cut off freq

$$p = 0$$

$$\Rightarrow \omega_c^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\Rightarrow \omega_c^2 = \frac{1}{\mu\varepsilon} \cdot \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\Rightarrow \omega_c = \frac{1}{\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\Rightarrow f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\pi^2\left[\left(\frac{m}{a}\right)^2 + \left(n/b\right)^2\right]}$$

$$\Rightarrow f_c = \frac{c}{2\pi} \cdot \pi \sqrt{\left(\frac{m}{a}\right)^2 + \left(n/b\right)^2} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\boxed{f_c = c/2 \sqrt{(m/a)^2 + (n/b)^2}}$$

$$\lambda_c = c/f_c$$

$$= \frac{c}{c/2\sqrt{(m/a)^2 + (n/b)^2}}$$

$$\boxed{\lambda_c = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}}}$$

This is the Expression for cut off wavelength

$$\lambda_c\, TE_{10} = \frac{2}{\sqrt{(1/a)^2}}$$

$$\lambda_c\, TE_{10} = 2a$$

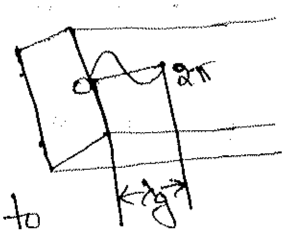
$$\lambda_c\, TE_{01} = 2b$$

$$\lambda_c\, TE_{20} = a$$

Note :— All the frequencies greater than f_c are propagated and freq's less than f_c are attenuated in the waveguide so the wave guide acts as highpass filter.

Guided wavelength (λ_g) :—

$$\lambda_g = \frac{2\pi}{\beta}$$



The guided wavelength is defined as the distance travelled by the wave in WG inorder to undergo a phaseshift of 2π radians

It is related to phase const β as

$$\lambda_g = \frac{2\pi}{\beta}$$

The guided wavelength λ_g is differ from freespace wavelength λ_0. The velocity of wave in waveguide (v) is always greater than the velocity of wave in freespace (c)

$$\therefore \quad \lambda_g > \lambda_0$$

$$v = \lambda_g f$$
$$c = \lambda_0 f$$

Relation among λ_g, λ_0 and λ_c :—

The guided wave length is defined as

$$\lambda_g = \frac{2\pi}{\beta}$$

At higher freq's propagation const. = phase const

$$P = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

$$j\beta = \sqrt{\omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon}$$

$$j\beta = \sqrt{-(\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon)}$$

$$= j\sqrt{(\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\lambda_g = \frac{2\pi}{\beta}$$

$$= \frac{2\pi}{\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{2\pi c}{2\pi f\sqrt{1-\left(f_c/f\right)^2}}$$

$$= \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$\because \; f_c = c/\lambda_c$

$f_0 = c/\lambda_0 \; ; \; \frac{f_c}{f_0} = \frac{\lambda_0}{\lambda_c}$

$$\boxed{\therefore \; \lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

$$\frac{\lambda_g}{\lambda_0} = \frac{1}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2} = \frac{\lambda_0}{\lambda_g} \Rightarrow 1 - \frac{\lambda_0^2}{\lambda_c^2} = \frac{\lambda_0^2}{\lambda_g^2}$$

$$\frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2} = \frac{1}{\lambda_g^2}$$

$$\boxed{\frac{1}{\lambda_0^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}}$$

This is the relation b/w λ_0, λ_c and λ_g

Dominant mode in rectangular waveguide :—

TE waves :—

$$\lambda_c = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}}$$

$$= \frac{2ab}{\sqrt{(mb)^2 + (na)^2}}$$

TE_{10} $m=1, n=0$

$\lambda_c TE_{10} = 2a, \quad f_c = c/2a$

for TE_{01}, $\lambda_c TE_{01} = 2b, \quad f_c = c/2b$

for TE_{20}, $\lambda_c TE_{20} = a, \quad f_c = c/a$

For TE_{02}, $\lambda_c TE_{02} = b, \quad f_c = c/b$

For TE_{11}, $\lambda_c TE_{11} = \frac{2ab}{\sqrt{a^2+b^2}}, \quad f_c = \frac{c\sqrt{a^2+b^2}}{2ab}$

Degenarative modes: When ever two of more modes have the same cut off frequency they are said to be degenarative modes.

In rectangular wave guides TE_{mn} and TM_{mn} modes are always degnarate.

For $a>b$, TE_{10} has lowest cut off frequency. ~~so it is the dominant mode TE_{10} also has~~ and highest cut off wavelength so it is called dominant mode in TE waves

TM waves :—

For TM_{11}, $\lambda_c TM_{11} = \frac{2ab}{\sqrt{b^2+a^2}}, \quad f_c = \frac{c\sqrt{a^2+b^2}}{2ab}$

For TM_{21}, $\lambda_c TM_{21} = \frac{2ab}{\sqrt{4b^2+a^2}}, \quad f_c = \frac{c\sqrt{4b^2+a^2}}{2ab}$

For TM_{12}, $\lambda_c TM_{12} = \frac{2ab}{\sqrt{b^2+4a^2}}, \quad f_c = \frac{c\sqrt{4a^2+b^2}}{2ab}$

TM_{11} has lowest cutoff frequency, highest cutoff wavelength So it is dominant mode in TM waves

Note :— TE_{10} is dominant mode in rectangular waveguides.

***phase velocity (vp) :—

phase velocity is defined as the ~~ratio of~~ Rate at which wave changes its phase in terms of guided wave length

$$v_p = \frac{\lambda_g}{\text{unit time}} = \lambda_g f$$

$$= \frac{\lambda_g f \cdot 2\pi}{2\pi}$$

$$= \frac{\omega}{2\pi/\lambda_g} = \frac{\omega}{\beta}$$

$$\boxed{\therefore v_p = \frac{\omega}{\beta}}$$

* phase velocity is greater than velocity of high seince

$$\boxed{\lambda_g > \lambda_0}$$

Any intelligence signal (or) modulation signal does not travels with velocity greater than velocity of light. So it is called phase velocity.

Expression for vp :—

$$P = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

For a propagating wave

$$P = j\beta = \sqrt{-(\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon)}$$

$$j\beta = j\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$\beta = \left(\sqrt{1-\frac{\omega_c^2}{\omega^2}}\right)\omega\sqrt{\mu\varepsilon}$$

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1-\frac{\omega_c^2}{\omega}}$$

phase velocity $v_p = \frac{\omega}{\beta}$

$$= \frac{\omega}{\omega\sqrt{\mu\varepsilon}\cdot\sqrt{1-\frac{\omega_c^2}{\omega^2}}}$$

$$= \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-\frac{f_c^2}{f^2}}}$$

$$\boxed{v_p = \frac{c}{\sqrt{1-\frac{f_c^2}{f^2}}}}$$

Group velocity (v_g) :—

The velocity of modulated wave in the waveguide is called group velocity and is given by

$$v_g = \frac{d\omega}{d\beta}$$

Expression for v_g :—

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$= \sqrt{\mu\varepsilon}\sqrt{\omega^2-\omega_c^2}$$

$$\frac{d\beta}{d\omega} = \sqrt{\mu\varepsilon}\cdot\frac{1}{2\sqrt{\omega^2-\omega_c^2}}\cdot 2\omega$$

$$= \omega\sqrt{\mu\varepsilon}\cdot\frac{1}{\sqrt{\omega^2-\omega_c^2}}$$

$$\frac{d\omega}{d\beta} = \frac{\sqrt{\omega^2-\omega_c^2}}{\omega\sqrt{\mu\varepsilon}}$$

$$\frac{d\omega}{d\beta} = \frac{\omega\sqrt{1-(\omega c/\omega)^2}}{\omega\sqrt{\mu\varepsilon}}$$

$$\therefore v_g = \frac{d\omega}{d\beta} = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\boxed{\therefore v_g = c\cdot\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_p v_g = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \cdot c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= c^2$$

$$\boxed{v_p v_g = c^2}$$

problems :-

1. Determine cut off wavelength for dominant mode in a rectangular waveguide of breadth 10cms for a 2.5 GHz signal calculate guided wavelength, phase velocity and group velocity

sol:- Given

$a = 10$ cms

$f = 2.5$ GHz

$\lambda_0 = c/f$

$$= \frac{3\times 10^{10}}{2.5\times 10^9}$$

$\lambda_0 = 12$ cms

For TE_{10}, $\lambda_c = 2a$

$= 2\times 10$

$= 20$ cms

$$\lambda g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$= \frac{12}{\sqrt{1-\left(\frac{12}{20}\right)^2}}$$

$$= 15\text{cms}$$

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$= \frac{3\times10^{10}}{\sqrt{1-\left(\frac{12}{20}\right)^2}} = 3.75\times10^8 \text{ m/sec}$$

(or) $v_p = \lambda g f = 15\times 2.5G = 3.75\times10^8$ m/sec

$$v_g = \frac{c^2}{v_p} = \frac{(3\times10^8)^2}{3.75\times10^8} = 2.4\times10^8 \text{ m/sec}$$

***

Wave Impedance :—

The wave impedance is defined as the ratio of the strength of the electric field in one transverse direction to the strength of magnetic field along other transverse direction. This ratio is termed as the wave impedance from a guide

$$\eta = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$$

$$= \frac{\sqrt{E_x^2+E_y^2}}{\sqrt{H_x^2+H_y^2}}$$

Impedance of TE wave :—

$$\eta_{TE} = \frac{E_x}{H_y} \quad \left[\eta_{TE} = \frac{\frac{+j\omega\mu}{k^2}\frac{\partial H_z}{\partial y}}{+j\beta/k^2 \; \partial H_z/\partial y} = \cancel{\frac{E_x}{H_y}}\right]$$

$$\eta_{TE} = \frac{\frac{j\omega\mu}{K^2} A\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}}{\frac{j\beta}{K^2} A\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}}$$

$$= \frac{\omega\mu}{\beta}$$

$$\eta_{TE} = \frac{-E_y}{H_x} = \frac{+\omega\mu}{\beta}$$

$$\eta_{TE} = \frac{\omega\mu}{\beta}$$

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$\eta_{TE} = \frac{\omega\mu}{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}$$

$$\eta_{TE} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}\sqrt{1-(f_c/f)^2}}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \cdot \frac{1}{\sqrt{1-(f_c/f)^2}}$$

$$\boxed{\eta_{TE} = \eta_0 \cdot \frac{1}{\sqrt{1-(f_c/f)^2}}}$$

Impedance of a TM wave :—

$$\eta_{TM} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\frac{j\beta}{K^2}\frac{\partial E_z}{\partial x}}{\frac{j\omega\varepsilon}{K^2}\frac{\partial E}{\partial x}}$$

$$= \frac{\beta}{\omega\varepsilon}$$

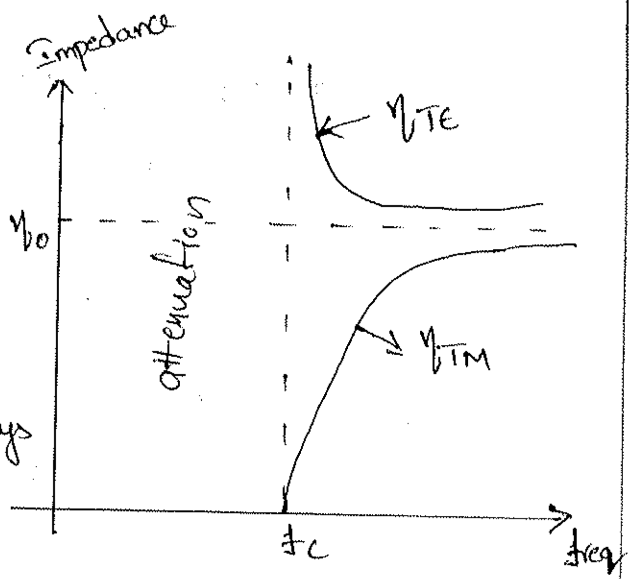
$$\eta_{TM} = \frac{\beta}{\omega\varepsilon}$$

$$\eta_{TM} = \frac{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}{\omega\varepsilon}$$

$$= \frac{\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c^2}{f^2}\right)}}{\omega\varepsilon}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \cdot \sqrt{1-\left(\frac{f_c^2}{f^2}\right)}$$

$$\boxed{\eta_{TM} = \eta_0\sqrt{1-(f_c/f)^2}}$$



$$\eta_{TE} \times \eta_{TM} = \eta_0^2$$

Impedance for TE wave is always greater than TM wave

$$\eta_{TE} > \eta_{TM}$$

* wave impedance for TM wave is always less than free space impedance

At $f = f_c$, $\eta_{TM} = 0$, $\eta_{TE} = \infty$

If $f < f_c$, the impedance is very high

problems :—

2. An air filled rectangular waveguide has dimensions of 0.9" x 0.4" supporting TE_{10} mode at a freq of 9800 MHz. calculate the percentage of change in the impedance for 10% increase in the operating freq

Sol:— Given

$f = 9800$ MHz

$$\eta_{TE} = \frac{\eta_0}{\sqrt{1-(f_c/f)^2}}$$

$$f_c = \frac{c/2}{\sqrt{(n/a)^2+(n/b)^2}} \cdot \frac{c}{2} \cdot \sqrt{(m/a)^2+(n/b)^2}$$

For TE_{10} mode, $f_c = \frac{c}{\cancel{2}\ 2} \left(\sqrt{(1/a)^2}\right)$

$$= \frac{c}{2a}$$

$1'' = 2.54$ cm

$$= \frac{c}{2(0.4)''}$$

$a = 0.9'' = 0.9 \times 2.54 = 2.286$ cm

$$f_c = \frac{c}{2(2.286)\text{cm}} = \frac{3\times10^{10}}{2(2.286)}$$

$$f_c = 6.56\ \text{GHz}$$

$$\eta_{TE} = \frac{377}{\sqrt{1-\left(\frac{6.56G}{9.8G}\right)^2}}$$

$$\eta_{TE} = 507.46\,\Omega$$

New freq $f' = 9800 + 9800 \times \frac{10}{100}$

$$= 10.78\ \text{GHz}$$

$$\therefore \eta_{TE} = \frac{377}{\sqrt{1-\left(\frac{6.56G}{10.78G}\right)^2}}$$

$$\eta_{TE} = 475\,\Omega$$

change in impedance $= 507.46 - 475$

$$= 32.46\,\Omega$$

$$= 32.46\,\Omega$$

$$\%\ \text{change} = \frac{32.46}{507.46} \times 100$$

$$= 63.9\% \cong 64\%$$

power loss in a rectangular waveguide

If the wave freq is less than cutoff freq losses are exist in rectangular waveguide, That attenuation loss is due to power dissipation within the waveguide walls and dielectric within the waveguide

If freq 'f' is less than f_c, the propagation const will have only attenuation const

$$P = \alpha + j\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

$$= \sqrt{\omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon}$$

$$P = \alpha = \sqrt{\omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon}$$

$$P = \alpha = \omega_c\sqrt{\mu\varepsilon}\sqrt{1 - (\omega/\omega_c)^2}$$

$$\alpha = \frac{2\pi f_c}{c}\sqrt{1 - (f/f_c)^2}$$

$$\boxed{\alpha = \frac{2\pi}{\lambda_c}\sqrt{1 - (f/f_c)^2}}\ \text{Neper/meter}$$

1 NP = 8.686

$$\alpha = \frac{54.6}{\lambda_c}\sqrt{1 - (f/f_c)^2}\ \text{dB/}\cancel{\text{length}}\text{meter}$$

*** power transmission in rectangular waveguide :-

power transmission in rectangular waveguide can be calculated by complex poynting theorem

According to ~~apply~~ a poynting theorem, power transmitted through waveguide is given by

$$P_{tr} = \frac{1}{2}\int_s (E \times H^*)\, ds$$

For lossless dielectric medium in waveguide the average power following through a rectangular waveguide is given by

$$P_{tr} = \frac{1}{2} \int_S \frac{|E|^2}{\eta} ds = \frac{1}{2} \eta \int_S |H|^2 ds$$

For TM wave

$$\eta_{TM} = \eta_0 \sqrt{1 - (f_c/f)^2}$$

$$P_{trTM} = \frac{1}{2} \eta_0 \sqrt{1 - (f_c/f)^2} \int_0^a \int_0^b |H|^2 dx\, dy$$

$$P_{trTM} = \frac{1}{2\eta_0 \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \int_0^a \int_0^b |E|^2 dx\, dy$$

For TE waves

$$\eta_{TE} = \eta_0 \Big| \sqrt{1 - (f_c/f)^2}$$

$$P_{trTE} = \frac{1}{2} \frac{\eta_0}{\sqrt{1 - (f_c/f)^2}} \int_0^a \int_0^b |H|^2 dx\, dy$$

$$P_{tr\,TE} = \frac{1}{2\eta_0} \sqrt{1 - (f_c/f)^2} \int_0^a \int_0^b |E|^2 dx\, dy$$

power transmission is more in TE waves compared to TM waves. So generally we prefer TE waves

Impossibility of TEM wave in rectangular and Circular wave guide :-

The wave that will propagate in hollow rectangular waveguide (or) cylinders have been divided into two sets

1. Transverse Electric wave which has no Z-component of E ($E_z = 0$)
2. Transverse magnetic wave which has no Z-component of H ($H_z = 0$)

TE and TM can propagate within rectangular (or) Circular (or) in cylindrical waveguides of any crosssection, but TEM wave has no axial Component of either E or H since TEM wave cannot propagate within single conductor waveguide.

If TEM wave Exists inside the waveguide the lines of H will be a closed loops. $(\nabla \cdot H = 0)$ and lies in a plane $\perp^{lar}$ to the z-axis. Now by Maxwell's Equations, magnetomotive force around Each of these closed loop must be equal to axial current. axial current through H loop will be ~~condition~~ conduction current ~~in~~ or displacement ~~the inner~~ ~~Conductor~~ Conduction current require inner conductor, But there will be no inner Conductor in hallow waveguides so the axial current must be displacement current

and ~~But~~ an axial displacement current require an axial Component of E. It is not present TEM wave

Therefore TEM wave Cannot Exist in a single Conductor waveguide

problems :—

3. A rectangular waveguide has Cross section of 1.5cm x 0.8 cm $\sigma = 0$, $\mu = \mu_0$, $\varepsilon = 4\epsilon_0$, the magnetic field component is given as $H_x = 2 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{3\pi y}{b}\right) \cdot \sin\left(\pi \times 10^{11} t - \beta z\right)$ determine mode of propagation, cut off freq phase constant, propagation const and wave impedance

Sol:— In TE and TM waves, common factor of magnetic field Component is

$$H_x = \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

Here $m=1$, $n=3$

So the mode is TE_{13} or TM_{13}

Cut off freq $f_c = ?$

$$f_c = c/2 \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(n/b\right)^2}$$

$$= \frac{1}{2\sqrt{\mu_0 \cdot 4\varepsilon_0}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(n/b\right)^2}$$

$$= \frac{1}{4\sqrt{\mu_0\varepsilon_0}} \sqrt{\left(\frac{1}{1.5}\right)^2 + \left(\frac{3}{0.8}\right)^2} \qquad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$= \frac{3\times10^{10}}{4} \sqrt{\left(\frac{1}{1.5}\right)^2 + \left(\frac{3}{0.8}\right)^2}$$

$$= 28.57\ GHz$$

phase constant, $\beta = ?$

$$2\pi f t = \pi \times 10^{11} t$$

$$f = 5\times10^{10}\ Hz$$

where 'f' is freq of wave

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1-(f_c/f)^2}$$

$$= 2\pi f\sqrt{\mu_0 \cdot 4\varepsilon_0}\sqrt{1-(f_c/f)^2}$$

$$= 4\pi f\sqrt{\mu_0\varepsilon_0}\sqrt{1-(f_c/f)^2}$$

$$= \frac{4\pi f}{c}\sqrt{1-(f_c/f)^2}$$

$$= 1718.81\ rad/m$$

propagation constant, $p = \alpha + j\beta$

$f > f_c$

$\therefore \; p = j\beta$

$= j1718.81$ rad/m

$$\eta_{TM} = \eta_0 \sqrt{1 - (f_c/f)^2}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \sqrt{1 - (f_c/f)^2}$$

$$= \sqrt{\frac{\mu_0}{4\varepsilon_0}} \sqrt{1 - (f_c/f)^2}$$

$$= \frac{1}{2}\eta_0 \sqrt{1 - (f_c/f)^2}$$

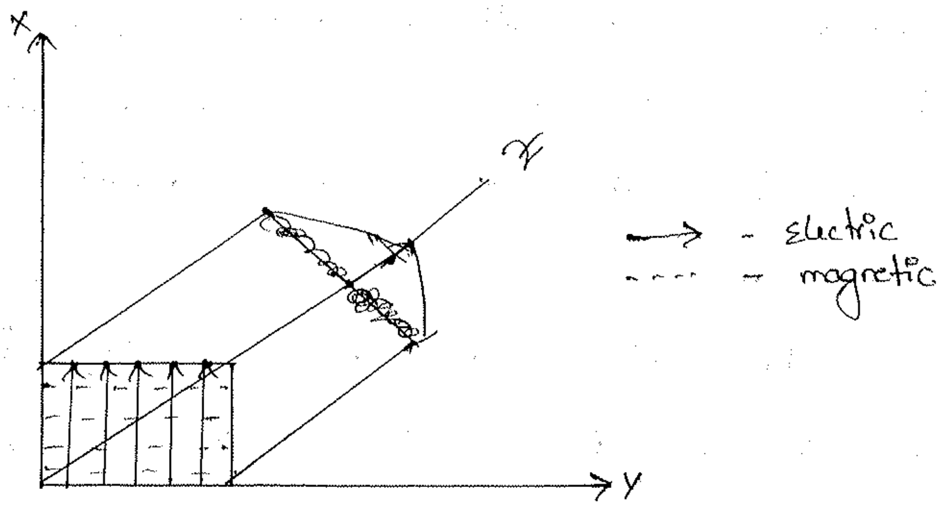
$$= 154.69\,\Omega$$

$$\eta_{TE} = \frac{\eta_0}{\sqrt{1 - (f_c/f)^2}}$$

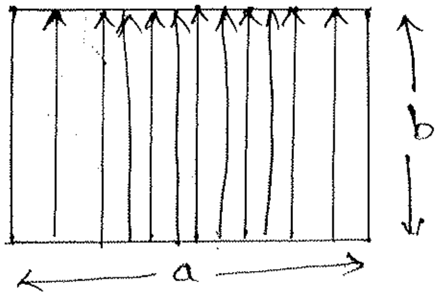
$$= \frac{\sqrt{\frac{\mu_0}{4\varepsilon_0}}}{\sqrt{1 - (f_c/f)^2}} = \frac{1}{2}\eta_0 \frac{1}{\sqrt{1 - (f_c/f)^2}}$$

$$= 229.68\,\Omega$$

*** Field patterns in rectangular waveguide :-

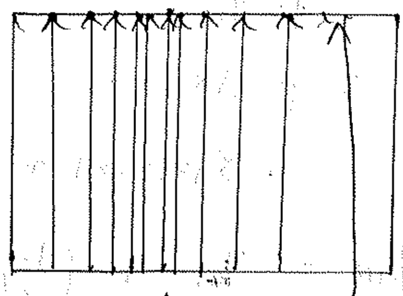


TE10

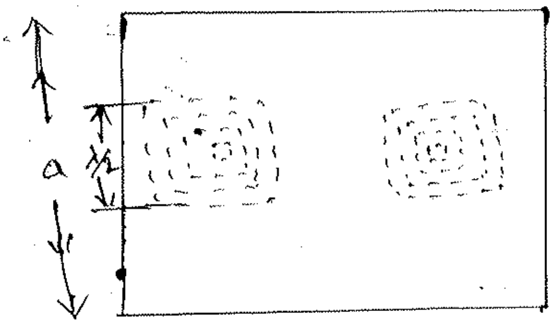


Front view

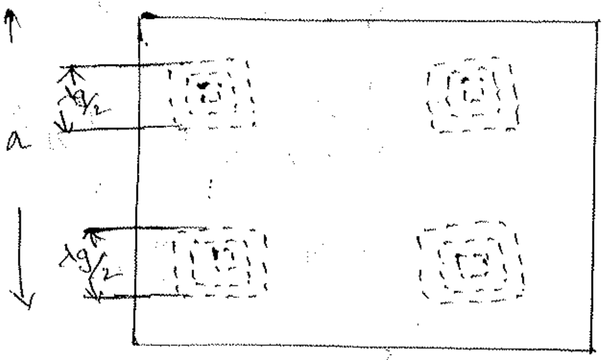
TE20



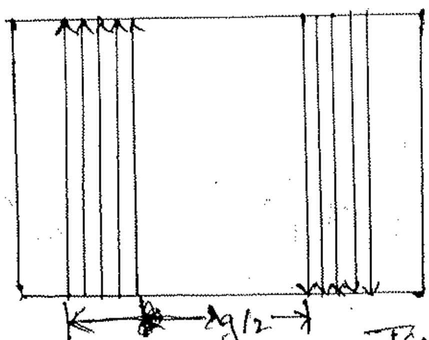
Front view



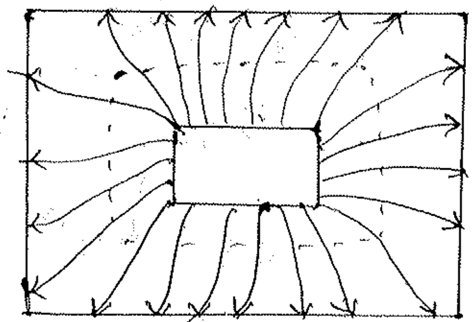
Top view



Top view



side view TE10, TE20



TE11 front view

we designate particular mode as TE_{mn} and TM_{mn} where m indicates no. of half wave ~~radiations~~ variations of Electric field (or) magnetic field across wider dimension 'a' lly 'n' indicates no of half wave variations of electric field (or) magnetic field across narrow dimension 'b'

The Electric field & magnetic field patterns in dominant mode TE_{10} is show in fig a) the Electric field lines Exists

(continuation in 14th page)

only at right angles to the direction of propagation where as magnetic field ~~has~~ has a component in the direction of propagation As well as $\perp^{lr}$ (or) normal to Electric field

The H field is in the form of closed loops ($\nabla \cdot H = 0$) which lies in a plane normal to E field i.e; parallel to ~~to~~ top & bottom of waveguide walls

The field pattern for TE_{20} mode is very similar to TE_{10} mode; but difference is two half wave ~~radiations~~ variations of E field & H field.

In TE_{11} mode, the E field & H field patterns are shown in fig : c)

problem :—

4. A rectangular waveguide has a = 4cm, b = 3 cm as it's sectional dimensions. Find all the modes which will propagate at 5000 MHz

Sol:-

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

For TE_{10}, $f_c = \frac{3\times10^{10}}{2}\sqrt{\left(\frac{1}{a}\right)^2} = \frac{3\times10^{10}}{2\times4}$

$f_c = 3.7$ GHz

$f_c < f$

For TE_{01}, $f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

$$= \frac{3\times10^{10}}{2\times3} = 5\text{ GHz}$$

$f = f_c$

TE_{01} mode is not propagated

For TE_{11} & TM_{11}

$$f_c = \frac{3\times10^{10}}{2}\sqrt{(1/a)^2+(1/b)^2}$$

$$= \frac{3\times10^{10}}{2}\sqrt{(1/4)^2+(1/3)^2}$$

$$= 6.25 \text{ GHz}$$

$f_c > f$

TM_{11} and TE_{11} does not propagated

For TE_{20}, $f_c = \frac{3\times10^{10}}{2}\sqrt{(2/4)^2} = 7.5 \text{ GHz}$

TE_{20} does not propagated

(5) The dimensions of waveguide are 2.5cm x 1cm the freq is 8.6 GHz find possible modes

Sol:- Given a = 2.5cm, b = 1cm

$f = 8.6$ GHz

For TE_{10} mode

$$f_c = \frac{c}{2}\sqrt{(m/a)^2+(n/b)^2}$$

$$= \frac{3\times10^{10}}{2}\sqrt{\left(\frac{1}{2.5}\right)^2}$$

$$= 6 \text{ GHz}$$

$f_c < f$

TE_{10} mode is propagated

For TE_{01} mode

$$f_c = \frac{c}{2b} = \frac{3\times10^{10}}{2\times1} = 15 \text{ GHz}$$

$f_c > f$

TE_{01} mode is not propagated

For TE_{11} & TM_{11}

$$f_c = \frac{c}{2ab}\sqrt{a^2+b^2}$$

$$= \frac{3\times10^{10}}{2\times2.5\times1}\sqrt{(2.5)^2+1^2}$$

$$= 16.15 \text{ GHz}$$

$$f_c > f$$

TE_{11} & TM_{11} modes does not propagate

For TE_{20}, $f_c = c/2\sqrt{(2/a)^2} = \frac{3\times10^{10}}{2}\sqrt{\frac{4}{2.5^2}}$

$$= 12 \text{ GHz}$$

TE_{20} does not propagate

Only TE_{10} mode is propagated.

(P) A rectangular wave guide ($a = 2cm$, $b = 1cm$) filled with de-ionized water ($\epsilon_r = 81$, $\mu_r = 1$) operates at 3 GHz. Determine all propagating modes and the corresponding cut off frequencies.

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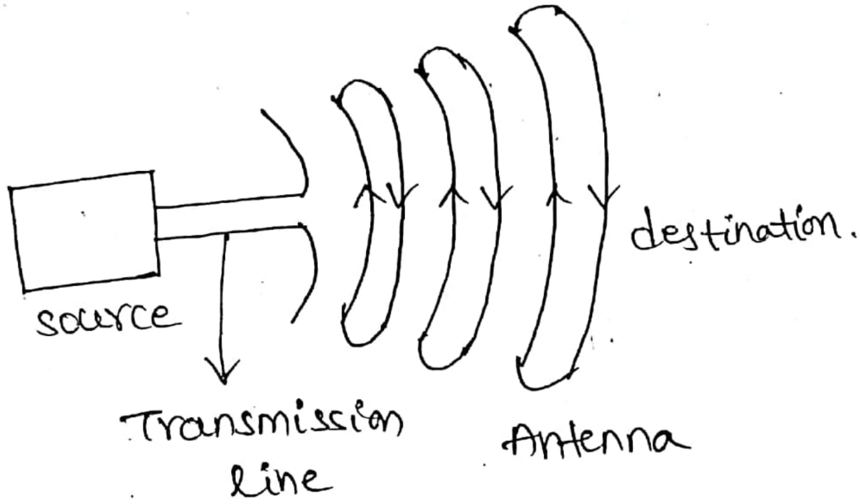
UNIT-1
ANTENNA FUNDAMENTALS

Antenna:- An Antenna is a metallic device which converts electrical signals to electromagnetic Waves and electromagnetic Waves to electrical signals.

→ An Antenna is in the form of a wire (or) rod which can be used as both Transmitting antenna and receiving antenna.

→ The first radio antenna was discovered by "Henrich hertz" in 1886.

EX:- Transmitting antenna, Receiving antenna, Cellsite antenna mobile antenna, Radio antenna.



Antenna Functions :-

1. Antenna acts as a Transducer
2. Antenna acts as an impedance matching device between Transmission and freespace.
3. It acts as a coupling device.
4. The antenna acts as a remote sensing, temperature measuring device.

Transm...

...pes of ...

i) dipole

Properties of Antenna :-

The antenna properties are applicable for Both Transmitting antenna, and Receiving antenna.

1. Equality of Impedances.
2. equality of effective lengths
3. Equality of directional patterns.

Antenna elements :-

1. Hertzian dipole (current element)
2. short dipole
3. short monopole
4. Halfwave dipole
5. quarter wave monopole

1. Hertzian dipole :- It is a basic linear antenna whose current distribution is constant. This is also called as "current element".

2. Short Dipole :- It is a basic linear antenna with a length is less than $\frac{\lambda}{4}$. The current distribution is triangular

3. short monopole :- It is a basic linear antenna with a length is less than $\frac{\lambda}{8}$. The current distribution is Triangular.

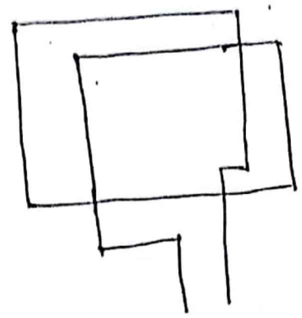
4. Half wave dipole :- It is a linear antenna, with a length is equal to $\frac{\lambda}{2}$. The current distribution is sinusoidal.

5. quarter wave monopole :- It is a linear antenna, with a length is equal to $\frac{\lambda}{4}$. The current distribution is sinusoidal.

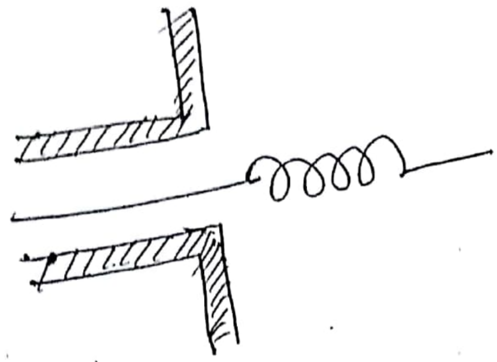
Types of Antennas :-

(i) dipole antenna

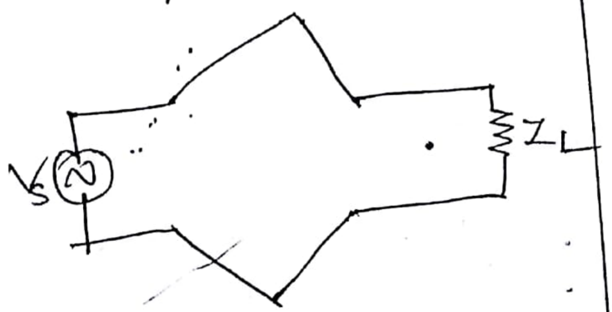
(ii) Loop antenna



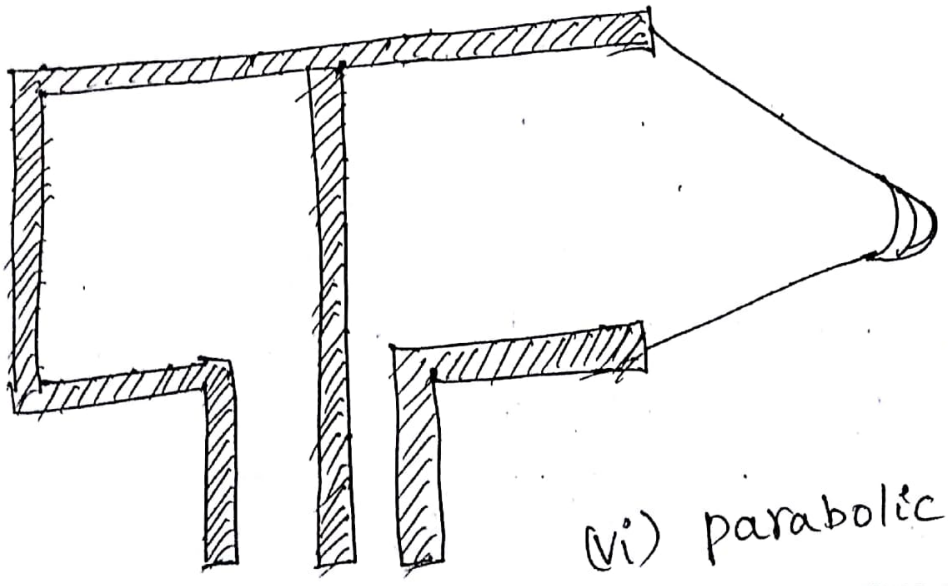
(iii) Helical antenna



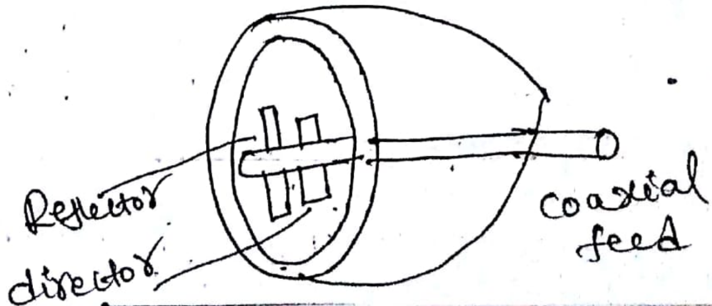
(iv) Rhombic antenna



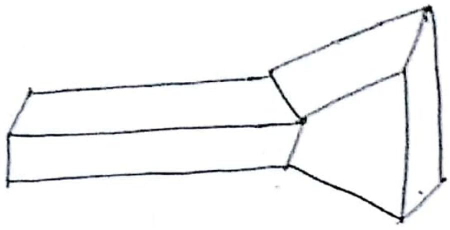
(V) dielectric antenna



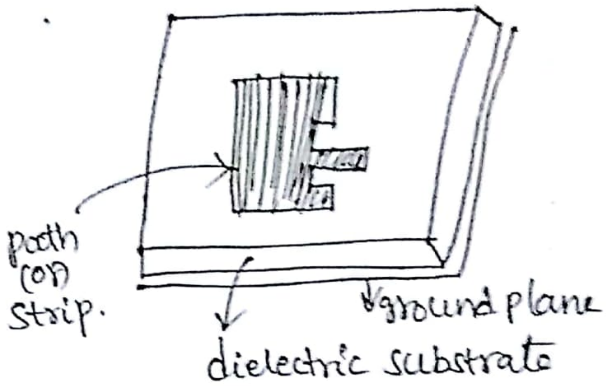
(Vi) parabolic antenna



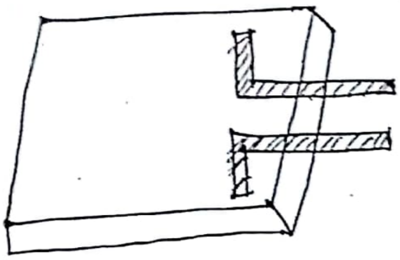
(vii) Horn antenna.



(viii) micro strip antenna



(ix) co planar antenna

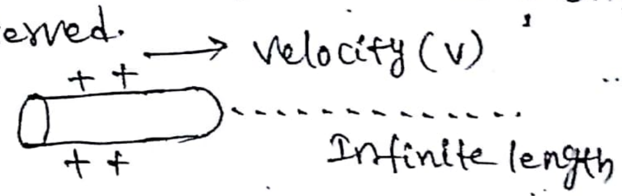


Radiation mechanism :-

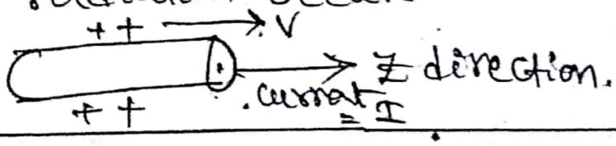
Radiation mechanism is the process of transmitting energy. The radiation occurs due to a source of electric charge.

(a) If a charge is static charge, then. there was No current generated. ∴ No radiation will be takes place

(b) If a charge is moving with a uniform velocity along the infinite length wire then only No radiation will be observed.



(c) When a pulse of charge is moving with a uniform velocity along a straight conductor in the z- direction. so the radiation occurs.



(∵ pulse of charge means finite length of charge)

Let us consider a charge per unit length is $\frac{q}{l}$ coulombs/m

The momentary current is $I = \frac{q}{l} \cdot \frac{dz}{dt} \rightarrow (1)$

Where $\frac{dz}{dt}$ is velocity 'V'

$\therefore I = \frac{q}{l} \cdot V \rightarrow (2)$

$\therefore I \cdot l = qV \rightarrow (3)$

differentiate eq (3) w.r.t 't' on both sides

$$l \frac{dI}{dt} = q \frac{dV}{dt}$$

($\because$ Acceleration $a = \frac{dV}{dt} = \frac{d}{dt}\left(\frac{dz}{dt}\right)$

$a = \frac{d^2z}{dt^2}$)

$$\Rightarrow \boxed{l \frac{dI}{dt} = qa}$$

(OR)

$$\boxed{l \frac{dI}{dt} = q \frac{d^2z}{dt^2}}$$

(or)

$$\boxed{\frac{dI}{dt} = \frac{qa}{l}}$$

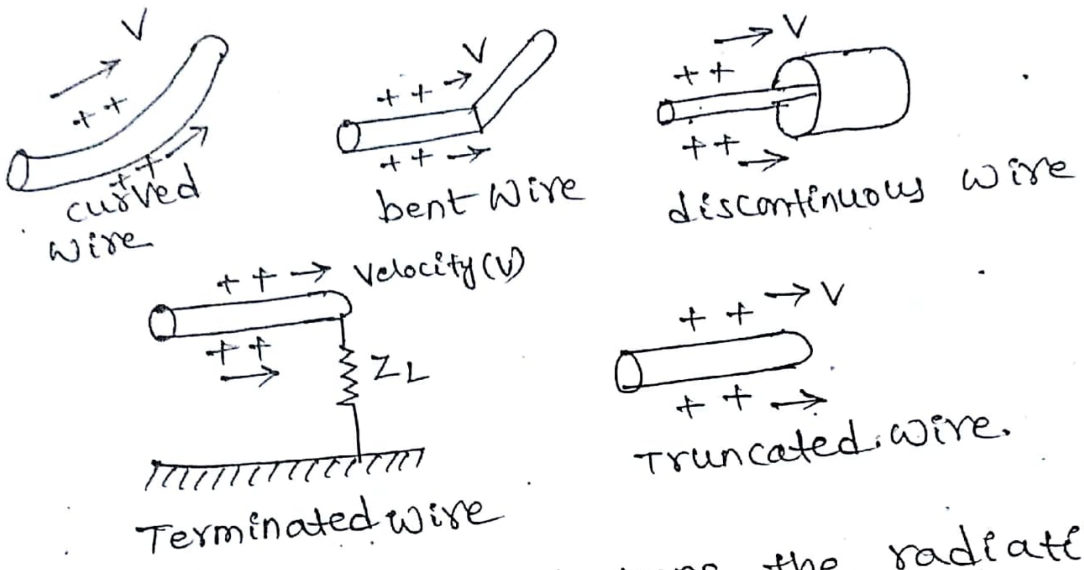
This equation represents a fundamental electro-magnetic radiation, that gives relationship between charge and current.

Radiation mechanism for single Wire :-

→ If a charge is stationary then there was no current will be generated. No radiation is occurs.

→ If a charge is moving with a uniform velocity along an Infinite length wire then only No radiation will be observed.

→ The radiation occurs only when a wire is curved, bent, discontinuous, terminated, truncated.



→ Due to the force of electrons the radiation is ~~pattern~~ Increased.

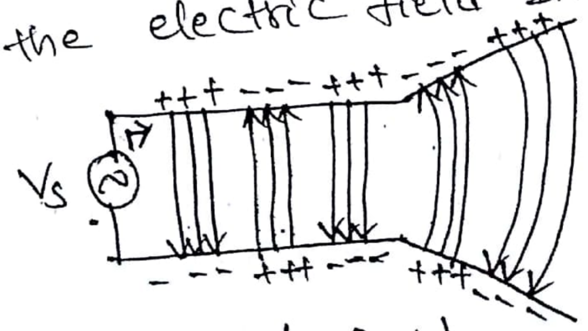
→ At the source end the Velocity is increased, at the destination end, Velocity is decreases.

→ Finally we conclude that the radiation is accelerated at source end and de-accelerated at destination end.

<u>Radiation Mechanism for Two Wires</u> :-

› When a Voltage source is applied, the electric field can be produced between two conductors (or) wires.

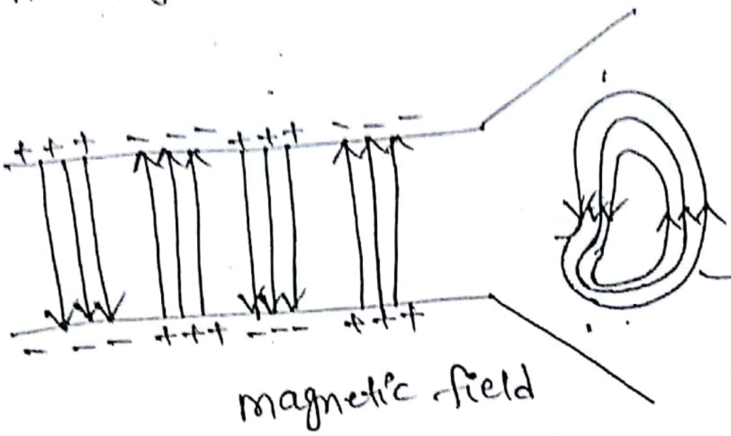
› The electric lines of force is parallel to the electric field that means the electric flux is directly proportional to the electric field Intensity



electric field

→ Due to the movement of charge carriers, the current will be produced, this current will generate a magnetic lines of force.

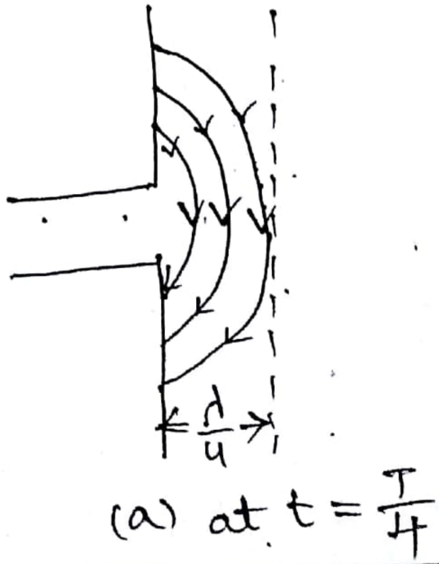
∴ The magnetic field forms the closed loops.



→ The electric lines travelling from positive charge carriers to negative charge carriers. While the magnetic lines form a closed loop.

Radiation Mechanism for dipoles:-

consider a small dipole is center in the first quarter period of time. (ie) $t = \frac{T}{4}$, at this time the charge gets a maximum value. Assume that the three electric lines, these lines are radially outwards at distance of $\frac{\lambda}{4}$.

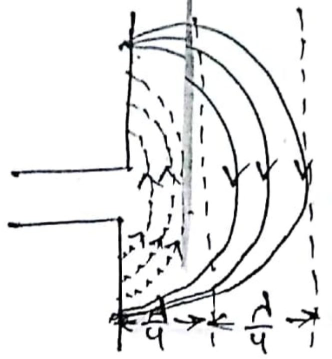


(a) at $t = \frac{T}{4}$

→ In the next qter period of time ($t = \frac{T}{4}$) the three electric es are produced at a distance of $\frac{d}{4}$. so the oppte charge lines are produced.

∴ The total he period is $t = \frac{T}{2}$, and total distance is $\frac{d}{2}$ ($\frac{d}{4}+\frac{d}{4}$) ... ... ($\because t = \frac{T}{4}+\frac{T}{4}$)

→ Due to the opsite charges, the charge density on the conducto is Zero. ∴ The charge is Neutral.

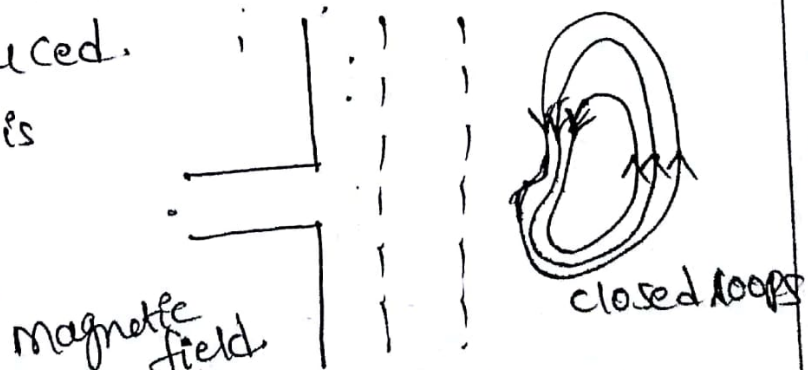


(b) at $t = \frac{T}{2}$ (ie first quarter and Second quarter period $t = \frac{T}{4}+\frac{T}{4} = \frac{T}{2}$)

→ finally We ~~conclu~~ conclude that the three electric lines are outward direction during the first quarter period of time, while the other three electric lines are in inward direction during second quarter period of time.

→ By Applying external force, these opposite charge lines are seperated by the conductor then the closed loops are produced.

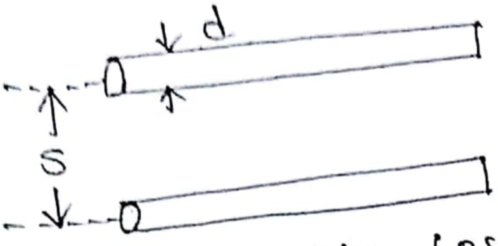
∴ The magnetic field is observed



Current distribution on thin linear wire antenna :- (5)

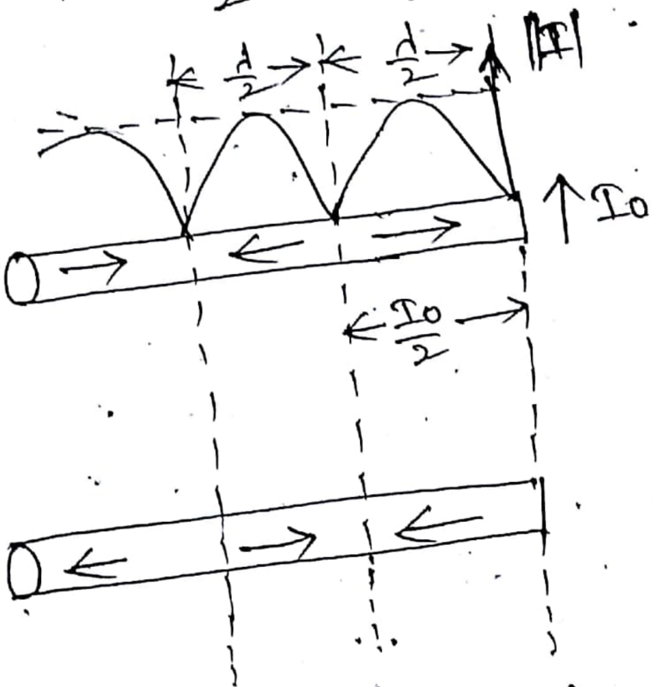
(i) For a two wire lossless Transmission line :-

Let us consider a two wire lossless Transmission line with the distance of seperation is 's' and diameter is 'd'.



(a) Two wire lossless Transmission line

When a free electrons are moving on the each conductor the travelling wave current is generated along each conductor. The magnitude of incident wave current is $\frac{I_0}{2}$.



(b) Current distribution for a two wire Transmission line.

→ At the end of each conductor the current will be reflected completely. The magnitude of this reflected current is also $\frac{I_0}{2}$ and phase shift is 180°.

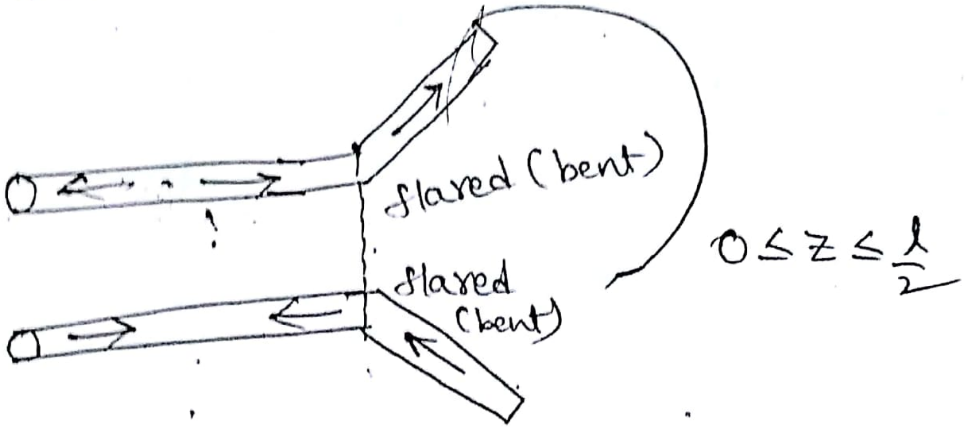
> When this reflected current is combined with a incident current, the standing wave pattern generated.

> In the adjecent half cycle the time period is varying. Also the current is varying.

∴ The radiation will be observed along each conductor.

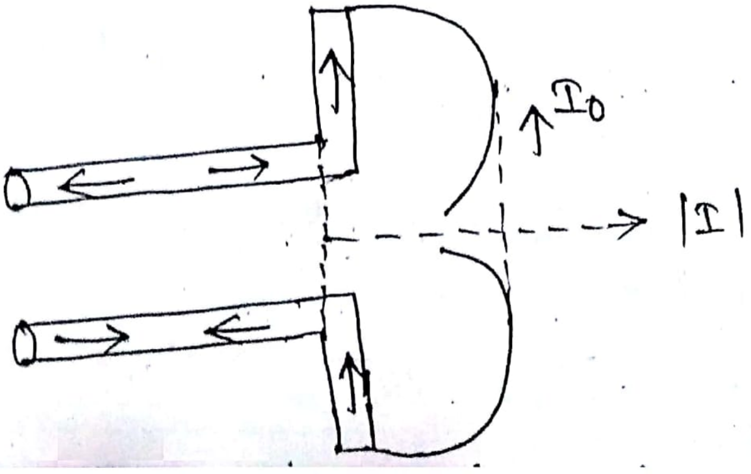
(ii) For a Flared Transmission line :-

If both the conductors between $0 \le z \le \frac{\lambda}{2}$ are bended (flared) then the current distribution will be no changes. (i.e) current distribution is same as in the first case. ∴ The radiation will be takes place.



(iii) For a Linear dipole :-

When a flared Transmission line is again bending, the linear dipole will be generated, this is called as dipole Antenna. dipole antenna also called as "Standing wave Antenna".



Iso-tropic Radiator (or) Hypothetical (or) fictitious radiator

→ Isotropic radiator is a radiator which radiates the energy in all directions uniformly. It can be used as reference antenna because the practical antenna can't radiate in all directions.

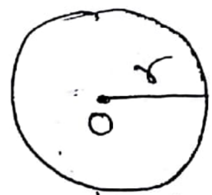
∴ Isotropic radiator is also used as Ideal Antenna.

→ Isotropic Radiator also called as hypothetical (or) fictitious radiator.

→ Consider an Isotropic radiator (Antenna) placed at the center of sphere with radius 'r'.

Let $\bar{P}$ be the poynting vector gives average power density

$$\therefore |\bar{P}| = P_r \rightarrow (1)$$



sphere
o = point source
r = radius.

The total power radiated is

$$P_{rad} = \iint |\bar{P}| \cdot d\bar{s} \qquad (\because |\bar{P}| = P_r)$$

$$\Rightarrow P_{rad} = \iint P_r \cdot d\bar{s} \rightarrow (2)$$

where $P_r = P_{avg}$ = average power density

$$\therefore P_{rad} = \iint P_{avg}\, d\bar{s}$$

$$= P_{avg} \iint d\bar{s} \qquad (\because \iint d\bar{s} = \text{surface area of sphere} = 4\pi r^2)$$

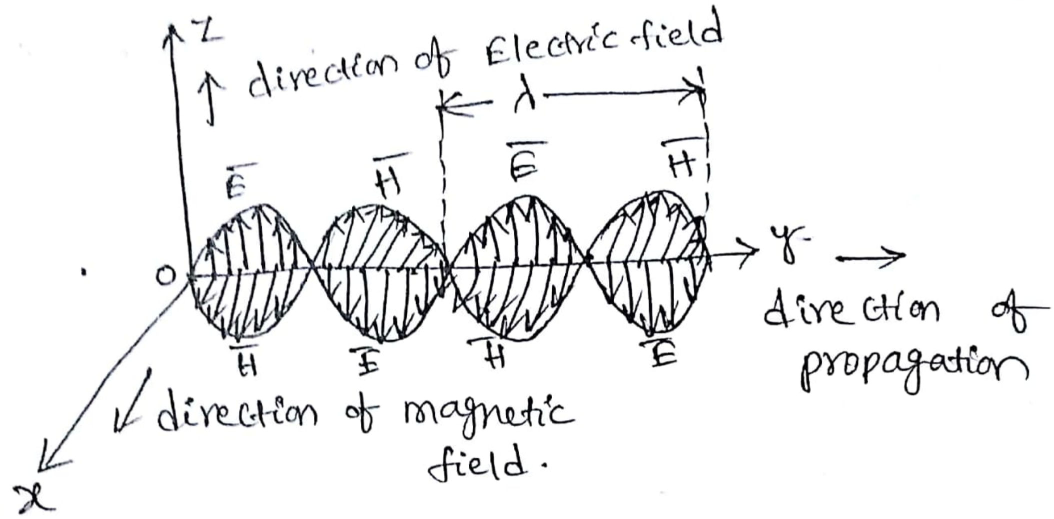
$$P_{rad} = P_{avg} \cdot 4\pi r^2$$

$$\therefore \boxed{P_{avg} = \frac{P_{rad}}{4\pi r^2}} \text{ Watts}/m^2$$

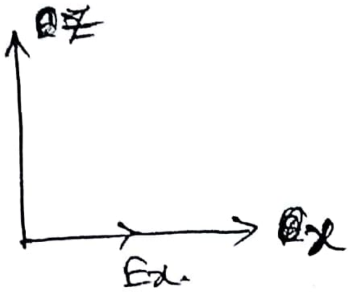
Polarization :- It is defined as to estimate the time varying behavior of the electric field strength.

(or)

The electric field is aligned with the one complete full cycle. There are three types of polarization.

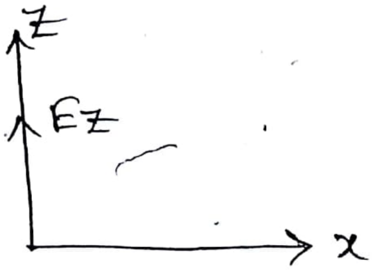


1. Linear polarization :- It is defined as the electro magnetic waves located in the complete space (or) total space.



Ex present
Ez Absent (zero)

(a) Horizontal polarization.



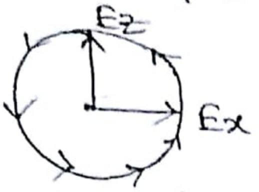
Ex Absent (zero)
Ez present

(b) Vertical polarization.

2. Circular polarization :-

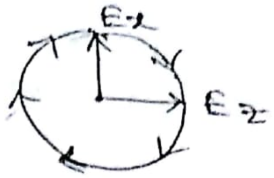
Two linear polarized waves having equal magnitudes and 90° phase shift then the wave is circularly polarized. (ie) $\boxed{E_x^2 + E_z^2 = E_a^2}$ (or) $\frac{E_x^2}{E_a^2} + \frac{E_z^2}{E_a^2} = 1$

Left circular polarization.



E_z leads E_x by 90°
($\phi = 90^\circ$)

Right Circular polarization



E_z lags E_x by 90°
($\phi = -90^\circ$)

(3) Elliptical polarization :- Two linear polarized waves having different magnitudes and 90° phase shift then the wave is said to be "elliptically polarized".

(ie) $$\boxed{\frac{E_x^2}{E_a^2} + \frac{E_z^2}{E_b^2} = 1}$$

Left elliptical polarization

Right elliptical polarization.

ANTENNA PARAMETERS

An Antenna is a basic element of communication system. It provides link between Transmitter to free space and free space to Receiver.

1. Radiation pattern
   (a) field radiation pattern
   (b) power radiation pattern.
2. Beam Width
3. Beam Area
4. Radiation Intensity
5. Directivity (or) maximum directive gain.

6. Power gain
7. Antenna Band width.
8. Beam efficiency
9. Antenna Aperture (effective area)
10. Effective length (effective height)
11. Antenna Temperature
12. Radiation efficiency.

Radiation pattern :- The radiation from Antenna can be measured in any direction interms of field strength.

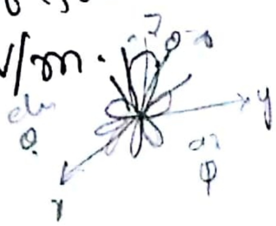
→ The field strength can be calculated by measuring voltages at two points on an electrical lines of force and then dividing by distance between two points.

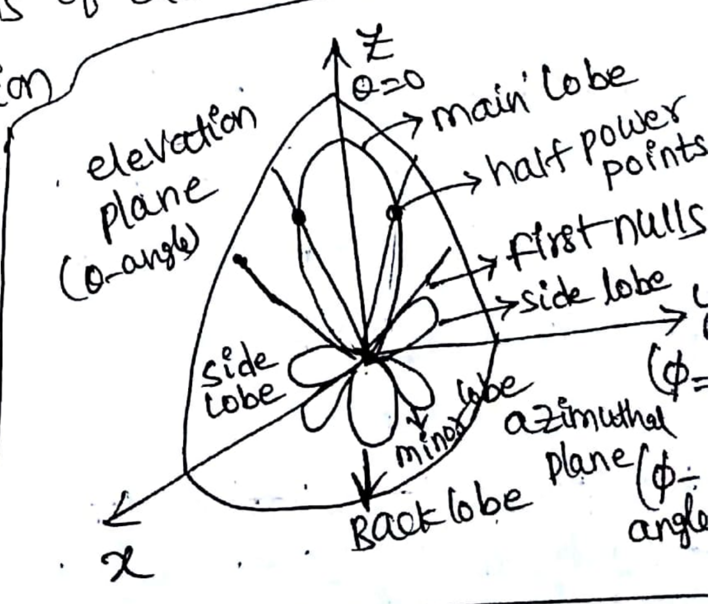
The radiation pattern can be classified into two types.
(a) field radiation pattern
(b) power radiation pattern.

Definition of Radiation pattern :- The radiation from an antenna is represented by graphically (or) Mathematically, interms of direction.

(a) field radiation pattern :- The field radiation pattern is defined as the radiation from antenna can be represented interms of electric field strength. $E(\theta,\phi)$. The field radiation is a graph which shows the direction of radiation.

→ The units of field radiation pattern are V/m.





→ It is a three dimensional pattern. So the spherical coordinate system is suitable
∴ The normalized field strength is given by

$$\boxed{E_{\theta_n} = \frac{E_\theta(\theta,\phi)}{E_\theta(\theta,\phi)_{max}}} \quad \boxed{E_{\phi_n} = \frac{E_\phi(\theta,\phi)}{E_\phi(\theta,\phi)_{max}}}$$

→ Where $E_\theta(\theta,\phi)$ is 'θ' component of electric field in the direction of θ and ϕ. $E_\phi(\theta,\phi)$ is ϕ component of electric field in the direction of θ and ϕ.

→ Normalized field pattern is defined as the ratio of field strength to its maximum value.

Main lobe :- It is a radiation lobe, which gives the maximum direction of radiation.

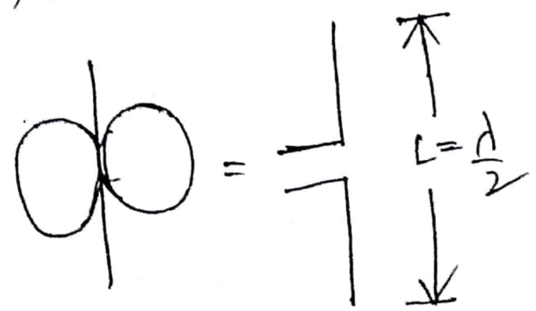
Side lobe :- Side lobes are lobes adjacent to the main lobe

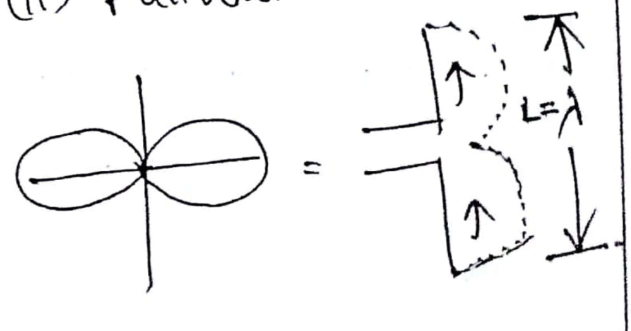
Minor lobe :- The lobes other than side lobes called as "minor lobes".

Back lobe :- The lobe opposite to the main lobe is called as back lobe. The angle between main lobe and ~~side~~ Back lobe is 180°.

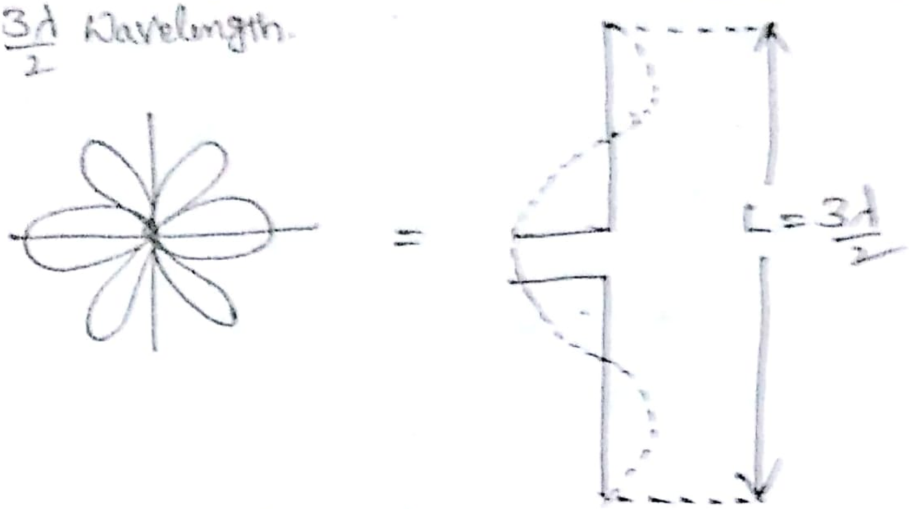
Examples of field strength pattern (Field Radiation pattern)

(i) Half Wavelength $(\frac{\lambda}{2})$. (ii) Full Wavelength (λ).

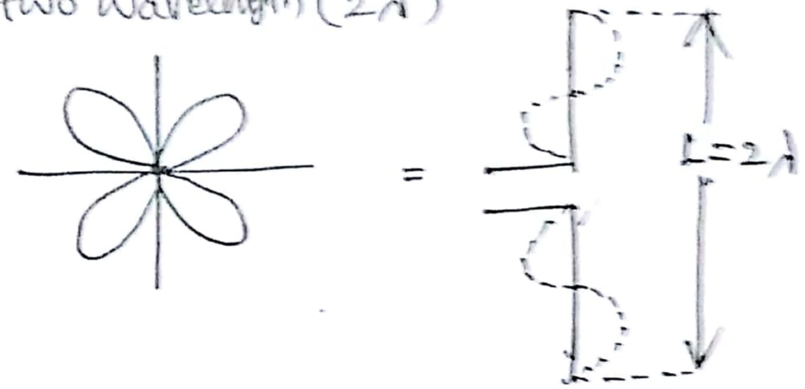




(iii) $\frac{3\lambda}{2}$ Wavelength.



(iv) two Wavelength (2λ)



(b) Power Radiation Pattern:-

→ Power radiation pattern is defined as the radiation of antenna can be represented interms of power per unit solid angle.

→ The power radiation pattern explained by power density. The power density is defined as power flow per unit area. It is given by $P_d(\theta, \phi)$

But we know that poynting vector

$$\bar{P} = \bar{E} \times \bar{H} \quad \text{(or)} \quad P = E \times H$$

$$= E \times \frac{E}{\eta_0}$$

$$P = \frac{E^2}{\eta_0}$$

($\because$ for free space $\frac{E}{H} = 120\pi = \eta_0$

$H = \frac{E}{\eta_0}$)

→ The power density is

$$\boxed{P_d(\theta,\phi) = \frac{1}{2}\frac{|E(\theta,\phi)|^2}{\eta_0}} \text{ Watts}/m^2$$

$\frac{1}{2}\frac{|E(\theta,\phi)|^2}{\eta_0}$

$E(\theta,\phi) = E_\theta^2(\theta,\phi) + E_\phi^2(\theta,\phi)$

Where $|E(\theta,\phi)| = \sqrt{E_\theta^2(\theta,\phi) + E_\phi^2(\theta,\phi)}$

η_0 = characteristic Impedance for free space

When $E(\theta,\phi)$ is maximum, the power density $P_d(\theta,\phi)$ is also maximum.

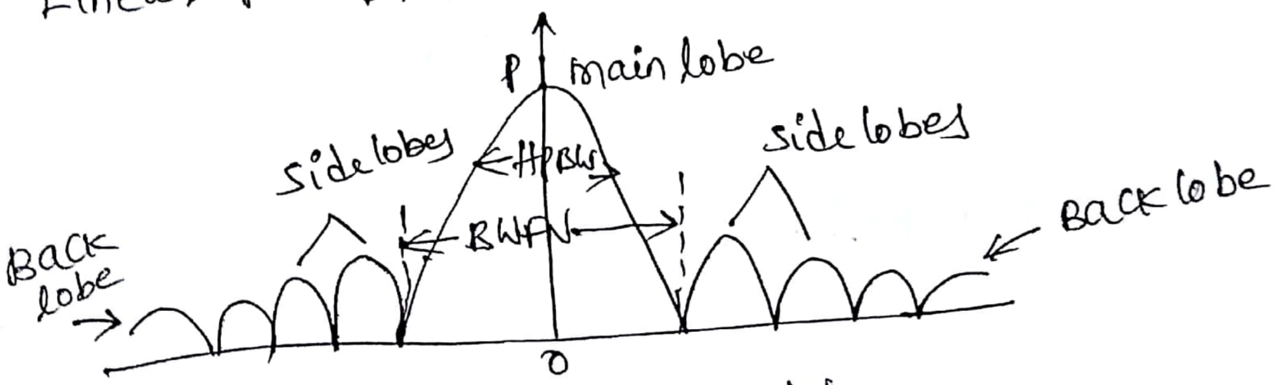
The normalized power pattern is given by

$$\boxed{P_{dn}(\theta,\phi) = \frac{P_d(\theta,\phi)}{P_d(\theta,\phi)_{max}}}$$

$$P_{dn}(\theta,\phi) = \frac{\frac{1}{2}\frac{|E(\theta,\phi)|^2}{\eta_0}}{\frac{1}{2}\frac{|E(\theta,\phi)|^2_{max}}{\eta_0}} = \frac{|E(\theta,\phi)|^2}{|E(\theta,\phi)|^2_{max}}$$

$$\therefore \boxed{P_{dn}(\theta,\phi) = f_n^2(\theta,\phi)}$$

Linear plot of power pattern.



HPBW = Half power Beam Width

BWFN = Beam Width between first nulls.

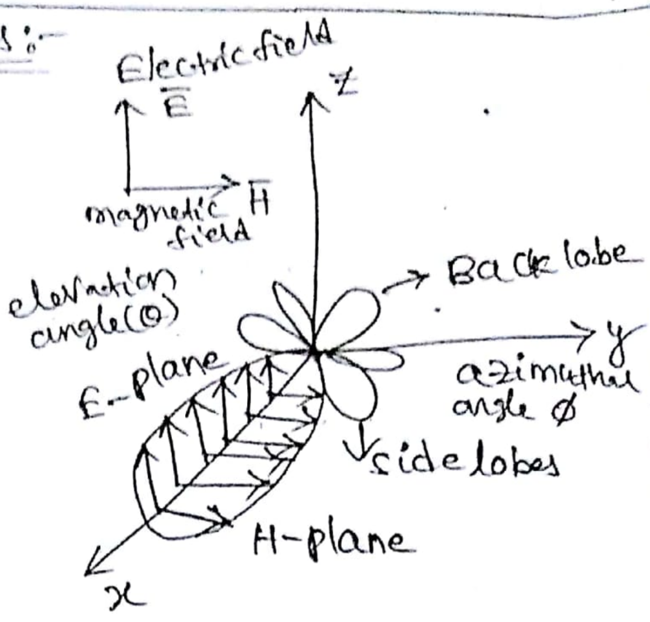
→ The power radiation pattern doesn't requires 3 dimensional approach. Because power radiation pattern is plane surfaces.

∴ For spherical surfaces only 3-dimensional pattern exists.

Patterns in principal planes:-

→ The performance of Antenna can be described interms of E-plane and H-plane. These planes are called as "principal" planes.

→ Generally principal plane patterns are two dimensional.



* **E-plane pattern**: It is defined as a plane consists of electric field vector ($\bar{E}$) and the direction of radiation is maximum.

→ It is also called as Vertical plane pattern.

→ E-plane exists in xz-plane.

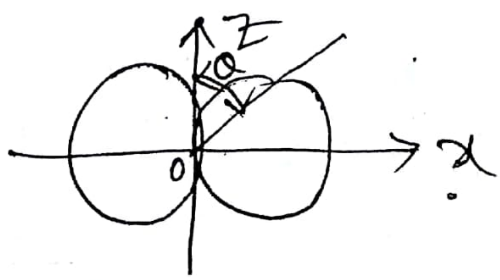
H-plane pattern :- It is defined as a plane consists of magnetic field vector ($\bar{H}$) and the direction of radiation is maximum.

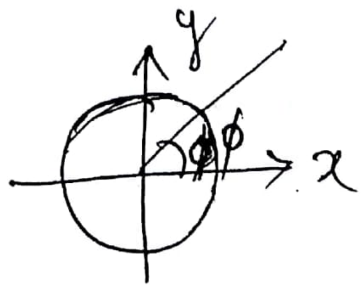
→ It is also called as Horizontal plane pattern.

→ the H-plane exists in xy-plane

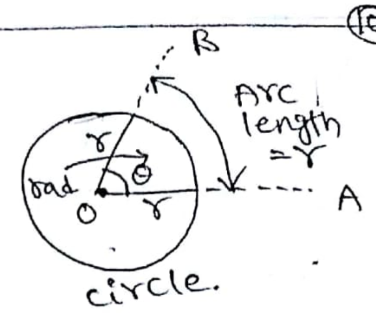
· the E-plane and H-planes are perpendicular to each other

Examples of principal planes





Radian and steradian:-



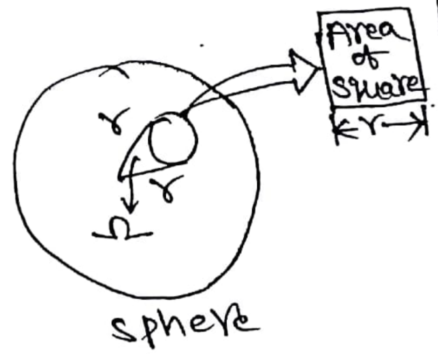
Radian :- The radian is simply a measure of plane angle. It can be defined as the plane angle with its vertex at the centre of the circle which can be extended by an arc (AB length) whose length is equal to 'r'.

The angle of complete circle is 2π radians (360°)

The circumference of circle is $2\pi r$.

steradian :- steradian is measure of solid angle. It is defined as the solid angle with its vertex at the centre of the sphere with radius 'r' which can be extended by area of sphere equivalent to area of square with each side is 'r'.

The area of sphere is $A = 4\pi r^2$.



$$1 \text{ steradian} = 1\,sr = \frac{\text{solid angle}}{4\pi}$$

and also $1\,sr = (1\,rad)^2 = (57.3\text{ deg})^2$

$$\boxed{1\,sr = 3283.3} \text{ square degrees}$$

$$4\pi\,sr = 4\pi \times 3283.3 \text{ square degrees}$$
$$= 41,259 \text{ square degrees}$$

The Infinitesimal area ds on sphere is

$$ds_r = (r d\theta)(r \sin\theta\, d\phi)$$

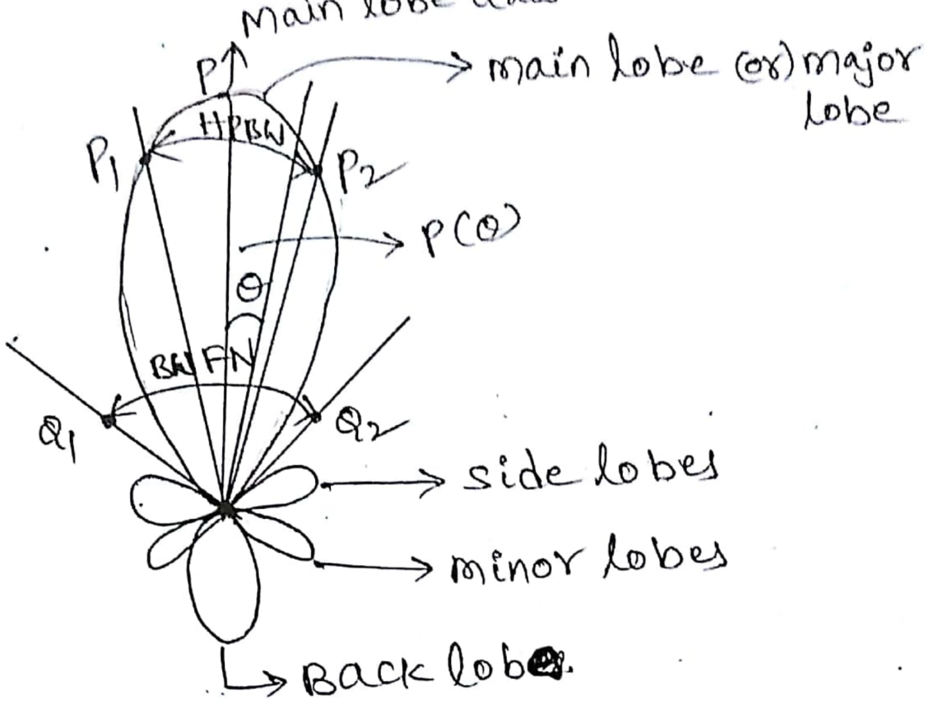
$$ds = ds_r = r^2 \sin\theta\, d\theta\, d\phi$$

∴ solid angle is

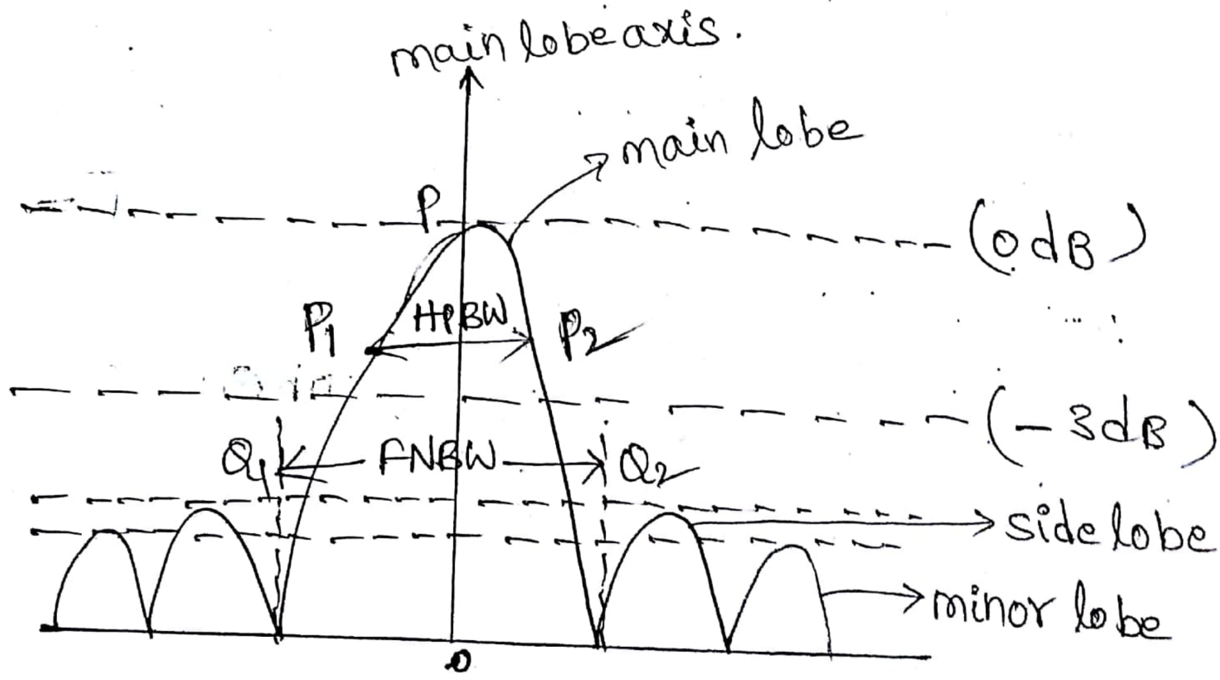
$$\boxed{d\Omega = \frac{ds}{r^2} = \sin\theta\, d\theta\, d\phi} \text{ steradian.}$$

Beam Width :-

The beam width is defined as the angular width in degrees between two point on a main lobe (or) major lobe of radiation pattern.



(a) Beam width on polar coordinates



(b) Beam Width on rectangular coordinates.

→ The Beam width is also called as half power beam width because at two half power points the power is reduced to half of its maximum power value.

→ The half power beam width is defined as the angular width in degrees between two half power points on the main lobe of radiation pattern.

→ It is also called as 3dB beam width from the above diagrams at point P, the power is maximum. At points P_1 and P_2 the power is reduces to half of its maximum power value.

BWFN :- (Beam width between First Nulls)

The angular width in degrees between two first nulls is called as first nulls beam width (or) Beam width between first nulls.

The directivity and Beam area can be related as

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{B} \rightarrow (1)$$

Where D = directivity

Ω_A = B = Beam area (or) Beam solid angle

= Beam width in E-plane x Beam width in H-plane.

= Beam width in vertical plane x Beam width in Horizontal plane

$$\therefore B = \Omega_A = \theta_E \times \phi_H \text{ steradians} \rightarrow (2)$$

$$D = \frac{4\pi}{\theta_E \times \phi_H} \text{ in steradians.}$$

$$D = \frac{4\pi}{\theta_E \times \phi_H}(57.3\,\text{deg})^2 = \frac{4\pi}{\theta_E \times \phi_H}(57.3)^2 \text{ degrees square}$$

$$\therefore \boxed{D = \frac{41,259}{\theta_E^\circ \times \phi_H^\circ}} \text{ square degrees.}$$

Beam Area (or) Beam solid angle :-

Beam area is defined as the Integral of normalized power patterns over the sphere. It is denoted by Ω_A. It is measured in steradians.

Beam area can be expressed as

$$\Omega_A = \oint_S P_{dn}(\theta,\phi)\, d\Omega \text{ steradians.}$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_{dn}(\theta,\phi)\, d\Omega \text{ steradians}$$

$$\boxed{\Omega_A = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_{dn}(\theta,\phi) \sin\theta\, d\theta\, d\phi} \text{ steradians.}$$

Where $d\Omega = \sin\theta\, d\theta\, d\phi$

Also Ω_A = HPBW in E-plane × HPBW in H-plane

= Beam width in vertical plane × Beam width in horizontal plane.

$$\boxed{\Omega_A = \theta_E \times \phi_H} \text{ steradians.}$$

Radian Intensity :- It is defined as the power per unit solid angle. It is denoted by 'U'.

Radiation Intensity can be expressed as

$$U(\theta,\phi) = r^2 P_d(\theta,\phi) \rightarrow (1)$$

The total power radiated is

$$P_{rad} = \oint_S P_d(\theta,\phi)\, ds$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_d(\theta,\phi)\, r^2 \sin\theta\, d\theta\, d\phi$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} [r^2 P_d(\theta,\phi)][\sin\theta\, d\theta\, d\phi]$$

$$\boxed{P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U(\theta,\phi)\,d\Omega \text{ steradians.}}$$

The average Radiation Intensity is

$$U_{avg} = r^2 P_a(\theta,\phi)_{avg}$$

$$= r^2 \cdot \frac{P_{rad}}{4\pi r^2}$$

$$\boxed{U_{avg} = \frac{P_{rad}}{4\pi}}$$

Beam efficiency :- Beam efficiency is defined as the ratio of power transmitted (or) received in one cone angle (θ_1) to the power transmitted (or) received by an antenna. The Beam efficiency also defined as

$$\varepsilon_M = \frac{\text{Main beam area}}{\text{Total beam area}} = \frac{\Omega_M}{\Omega_A} \longrightarrow (1)$$

Where Ω_M = main beam area

Ω_A = Total beam area.

$\Rightarrow \Omega_A = \Omega_M + \Omega_m \rightarrow (2)$ Where Ω_m = minor lobe area

Dividing equation (2) by Ω_A on both sides

$$\frac{\cancel{\Omega_A}}{\cancel{\Omega_A}} = \frac{\Omega_M + \Omega_m}{\Omega_A} \Rightarrow \frac{\Omega_M}{\Omega_A} + \frac{\Omega_m}{\Omega_A} = 1$$

$$\therefore \boxed{\varepsilon_M + \varepsilon_m = 1}$$

Where $\varepsilon_M = \frac{\Omega_M}{\Omega_A}$ = Beam efficiency

$\varepsilon_m = \frac{\Omega_m}{\Omega_A}$ = Stray factor.

Gain(G) :- The gain is defined as the ratio of maximum radiation intensity from Test Antenna (practical antenna) to the maximum radiation intensity from the reference antenna (Ideal Antenna). It is denoted by G.

$$\boxed{G = \frac{U_{max}}{U_0}}$$

The gain is measured in dB.

→ generally Isotropic antenna used as reference antenna

Directive Gain (G_D) :- The directive gain is defined as the ratio of radiation intensity in particular direction (θ, ϕ) to the average radiation intensity.

It is denoted by G_D.

$$\boxed{G_D = \frac{U(\theta, \phi)}{U_{avg}}}$$

(or)

Directive gain is also defined as the ratio of power density in particular direction (θ, ϕ) to the average power density.

(ie) $$\boxed{G_D = \frac{P_d(\theta, \phi)}{P_{avg}}}$$

$$G_D = \frac{P_d(\theta, \phi)}{\frac{P_{rad}}{4\pi r^2}}$$

$\{U(\theta, \phi) = r^2 P_d(\theta, \phi)\}$

$$G_D = P_d(\theta, \phi) \times \frac{4\pi r^2}{P_{rad}}$$

$$G_D = \frac{4\pi [r^2 P_d(\theta, \phi)]}{P_{rad}} = \frac{4\pi \times U(\theta, \phi)}{P_{rad}}$$

$\left(\because U_{avg} = \frac{P_{rad}}{4\pi}\right)$

$$G_D = \frac{U(\theta, \phi)}{\left(\frac{P_{rad}}{4\pi}\right)} \Rightarrow \boxed{G_D = \frac{U(\theta, \phi)}{U_{avg}}}$$

Directivity (D) :- The directivity is defined as the ratio of maximum radiation Intensity to the average radiation Intensity. It is denoted by D.

$$\boxed{D = G_{Dmax} = \frac{U(\theta,\phi)_{max}}{U_{avg}}}$$

(OR)

The directivity is defined as the ratio of maximum power density to the average power density. It is denoted by 'D'. It can be expressed as

$$D = G_{Dmax} = \frac{P_d(\theta,\phi)_{max}}{P_{avg}}$$

$$= \frac{P_d(\theta,\phi)_{max}}{\left(\frac{P_{rad}}{4\pi r^2}\right)} \qquad \left(\because P_{avg} = \frac{P_{rad}}{4\pi r^2}\right)$$

$$= P_d(\theta,\phi)_{max} \times \frac{4\pi r^2}{P_{rad}}$$

$$\Rightarrow D = \frac{4\pi \times r^2 P_d(\theta,\phi)_{max}}{P_{rad}} \qquad \left(U(\theta,\phi)_{max} = r^2 P_d(\theta,\phi)_{max}\right)$$

$$\Rightarrow \boxed{D = \frac{4\pi \times U(\theta,\phi)_{max}}{P_{rad}}}$$

$$D = \frac{U(\theta,\phi)_{max}}{\left(\frac{P_{rad}}{4\pi}\right)} \qquad \left(\because U_{avg} = \frac{P_{rad}}{4\pi}\right)$$

$$\boxed{D = \frac{U(\theta,\phi)_{max}}{U_{avg}}}$$

The directivity is also defined as maximum directive Gain. (G_{Dmax}).

Relation between directivity (D) and Beam area (Ω_A)

We have to show that $\boxed{D = \frac{4\pi}{\Omega_A}}$.

proof :- We know that directivity $D = \frac{U(\theta,\phi)_{max}}{U_{avg}}$

$$\Rightarrow D = \frac{U(\theta,\phi)_{max}}{\frac{P_{rad}}{4\pi}} = \frac{4\pi \times U(\theta,\phi)_{max}}{P_{rad}} \qquad \left(\because U_{avg} = \frac{P_{rad}}{4\pi}\right)$$

$$\Rightarrow D = \frac{4\pi \times r^2 P_d(\theta,\phi)_{max}}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U(\theta,\phi)\, d\Omega \text{ steradians}} \qquad \left(\because P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U(\theta,\phi) \times d\Omega\right)$$

$$\Rightarrow D = \frac{4\pi r^2 P_d(\theta,\phi)_{max}}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} r^2 P_d(\theta,\phi)\, d\Omega \text{ sr}} \qquad \left(\because r^2 P_d(\theta,\phi) = U(\theta,\phi)\right)$$

$$\Rightarrow D = \frac{4\pi \cancel{r^2} P_d(\theta,\phi)_{max}}{\cancel{r^2}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_d(\theta,\phi)\, d\Omega \text{ sr}}$$

$$\Rightarrow D = \frac{4\pi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \left[\frac{P_d(\theta,\phi)}{P_d(\theta,\phi)_{max}}\right] d\Omega \text{ sr}}$$

$$\Rightarrow D = \frac{4\pi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_{dn}(\theta,\phi)\, d\Omega \text{ steradians}} \qquad \left(\because \Omega_A = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_{dn}(\theta,\phi) \times d\Omega\right)$$

$$\therefore \boxed{D = \frac{4\pi}{\Omega_A}}$$

Hence proved

(14)

Resolution :- It is defined as half of the Beam width between first nulls.

$$\text{Resolution} = \frac{BWFN}{2} = \frac{FNBW}{2}$$

Also $HPBW = \frac{BWFN}{2}$

Front to Back Ratio :- It is defined as the ratio of power Transmitted in desired direction to the power transmitted in reverse direction.

$$FBR = \frac{\text{power Transmitted in desired direction}}{\text{power Transmitted in reverse direction}}$$

Antenna Band width :- Band width is defined as the difference between two band of frequencies. It is denoted by $\Delta\omega$ (or) Δf.

$$\text{Band width} = \Delta\omega = \omega_2 - \omega_1 = \frac{\omega_0}{Q}$$

$$\text{Band width} = \Delta f = f_2 - f_1 = \frac{f_0}{Q}$$

Power gain (G_P) :- power gain is defined as the ratio of power density in particular direction to the actual i/p power.

$$G_p = \frac{P_d(\theta,\phi)}{P_{in}} = \frac{\text{Power density in } (\theta,\phi)}{\text{Actual i/p power}}$$

The relation between G_P and G_D is given by

$$\boxed{G_P = \eta_r G_D}$$

(or)

$$G_{Pmax} = \eta_r G_{Dmax}$$

$$\boxed{G = \eta_r D}$$

Radiation efficiency (η_r)

It is defined as the ratio of power radiated to the actual input power.

We can express Prad interms of Pin

(ie) $P_{rad} = \eta_r P_{in}$.

$$\boxed{\eta_r = \frac{P_{rad}}{P_{in}}}$$

where P_{in} = actual input power

$= P_{rad} + P_{Loss}$

$$\boxed{\eta_r = \frac{P_{rad}}{P_{rad} + P_{Loss}}}$$

$$P_{rad} = I^2 R_{rad}$$

$$P_{Loss} = I^2 R_{Loss}$$

$$\eta_r = \frac{I^2 R_{rad}}{I^2 R_{rad} + I^2 R_{Loss}} = \frac{\cancel{I^2} R_{rad}}{\cancel{I^2}[R_{rad} + R_{Loss}]}$$

$$\boxed{\eta_r = \frac{R_{rad}}{R_{rad} + R_{Loss}}}$$

Relation between G_{Pmax} and G_{Dmax} :-

The maximum power gain is

$$G_{Pmax} = \frac{U(\theta,\phi)_{max}}{\left(\frac{P_{in}}{4\pi}\right)} \rightarrow (1)$$

The maximum directive gain is

$$G_{Dmax} = \frac{U(\theta,\phi)_{max}}{U_{avg}} = \frac{U(\theta,\phi)_{max}}{\left(\frac{P_{rad}}{4\pi}\right)} \rightarrow (2)$$

from eq (1), (2)

$$\frac{G_{Pmax}}{G_{Dmax}} = \frac{\cancel{U(\theta,\phi)_{max}} / \left(P_{in}/4\pi\right)}{\cancel{U(\theta,\phi)_{max}} / \left(\frac{P_{rad}}{4\pi}\right)}$$

$$\Rightarrow \frac{G_{Pmax}}{G_{Dmax}} = \frac{1}{\left(\frac{P_{in}}{4\pi}\right)} \times \frac{P_{rad}}{4\pi}$$

$$= \frac{\cancel{4\pi}}{P_{in}} \times \frac{P_{rad}}{\cancel{4\pi}}$$

$$= \frac{P_{rad}}{P_{in}}$$

$$= \eta_r$$

$$\therefore \boxed{G_{Pmax} = \eta_r G_{Dmax}}$$

$$\boxed{G_P = \eta_r G_D}$$

Antenna Aperture (Ae) :- (effective Aperture, capture area, effective area)

It is defined as the ratio of power received at the antenna load terminal to the poynting vector of the antenna. It is denoted by Ae

→ It is also called as effective aperture (or) effective area, capture area.

$$\boxed{A_e = \frac{P_{received}}{\bar{P}}}\ m^2 \qquad (\because \bar{P} = \text{poynting Vector} = \bar{E}\times\bar{H})$$

Aperture efficiency (η_a) :-

It is defined as the ratio of effective aperture to the physical aperture. It is denoted by η_a.

$$\boxed{\eta_a = \frac{\text{effective aperture}}{\text{physical aperture}} = \frac{A_e}{A_p}}$$

$$\boxed{\%\,\eta_a = \frac{A_e}{A_p}\times 100}$$

Effective Height :- (Leff (or) effective Length)

It is defined as the ratio of Induced voltage at receiving antenna to the under open ckt condition Incident electric field Intensity. It is denoted by Leff.

$$\boxed{L_{eff} = \frac{V_{oc}}{E}}\ \text{meters}$$

$$L_{eff} = \frac{\text{Induced Voltage under open ckt}}{\text{Incident Electric field Intensity}}$$

III - ECE
S1 & S2
A. Narasimha Reddy

UNIT-3

ANTENNA ARRAYS :- [redacted]

AWr ①

Antenna array :- The antenna array is a radiating system, in which the group of antennas are arranged in parallel to each other. Therefore to get the maximum radiation and the high directivity, Increased field strength.

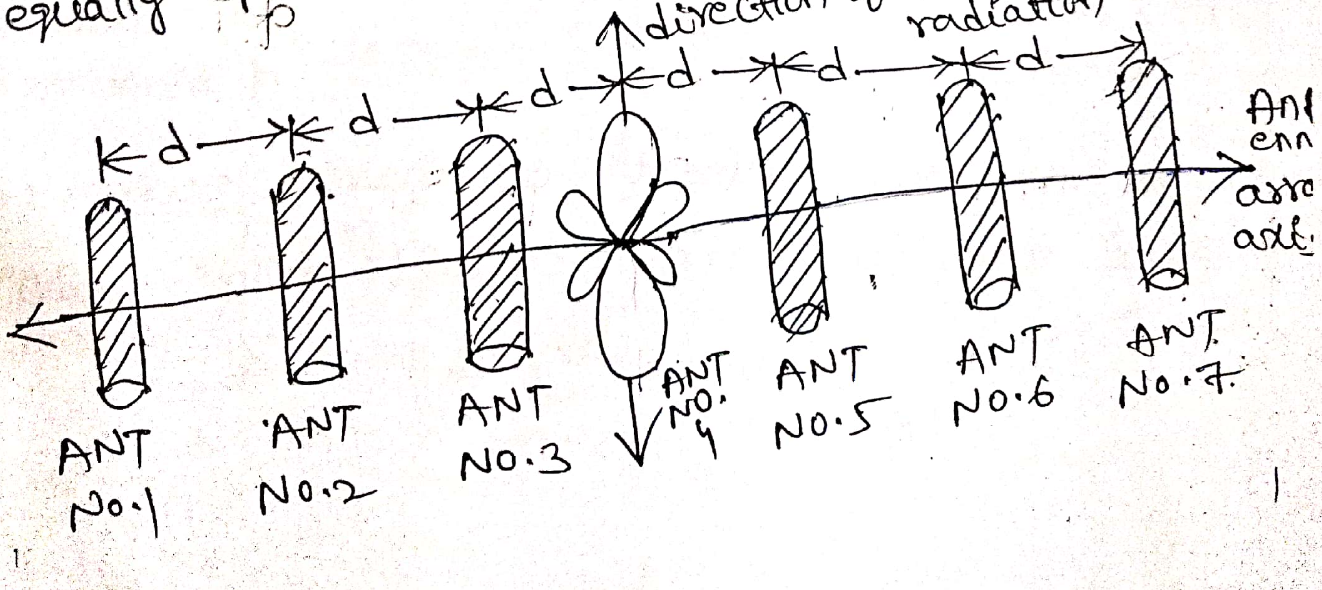
Various types of arrays :- There are different types of arrays.

(1) Broad side array
(2) End fire array
(3) collinear array
(4) parasitic array.

(1) Broad side array :-

→ Broad side array is defined as " An arrangement in which the principal direction of radiation is perpendicular to the array axis and also to the antenna plane.

→ It consists of Identical parallel antennas equally spaced along a line perpendicular to axes.



→ A horizontal radiation pattern is obtained when these arrays are vertically arranged.

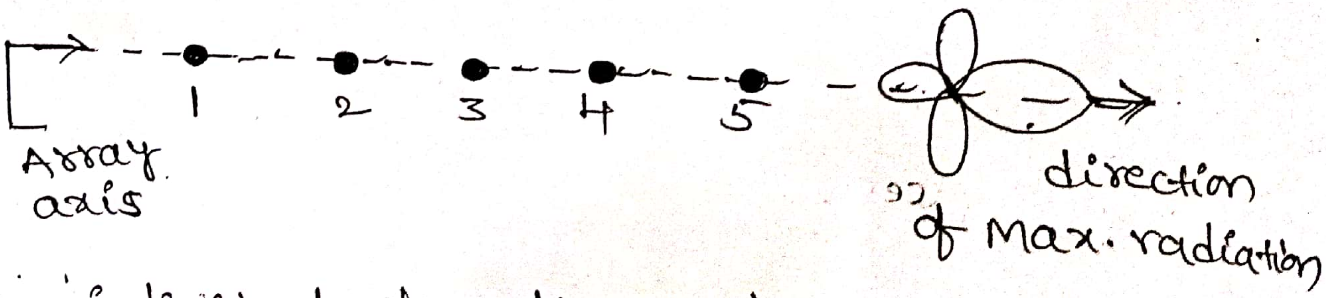
→ A vertical radiation pattern is obtained when these arrays are horizontally arranged.

→ In this broad side array, the individual elements are having currents of equal Amplitudes and same phases.

(2) End fire array :-

→ An end-fire array is defined as "The arrangement in which the principal direction of maximum radiation coincides with the direction of array axis.

→ The end fire array is similar to broad-side array except that individual elements are fed in with currents out of phase 180°.

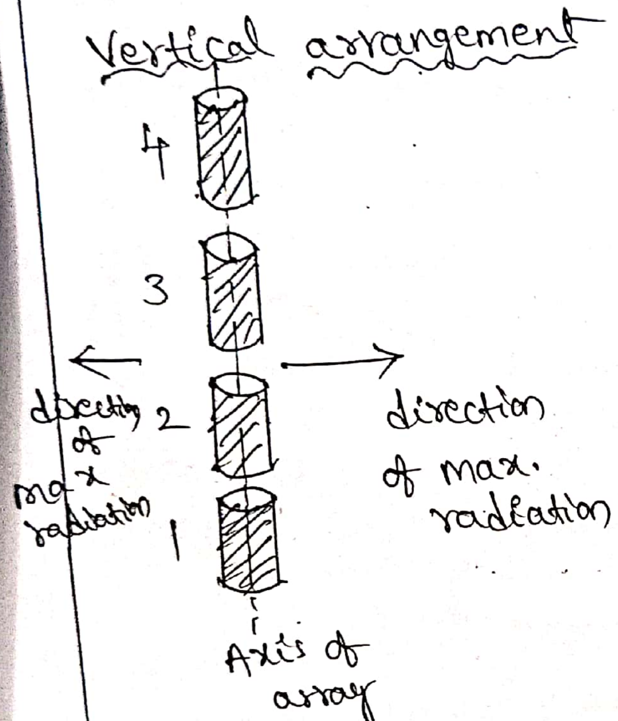


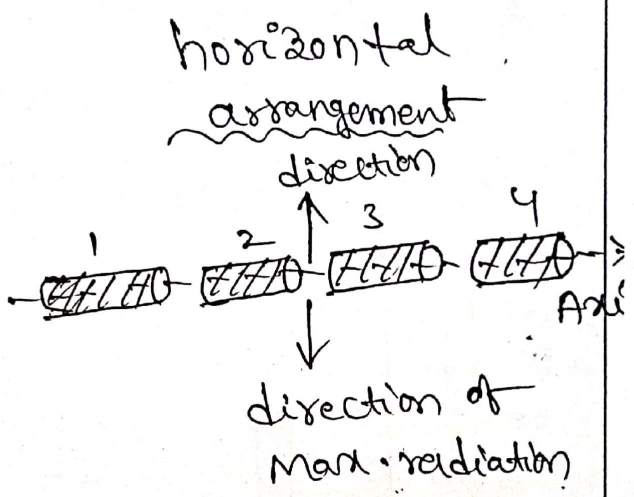
→ The individual elements are having currents of equal amplitudes and opposite phase (180°)

Collinear arrays :-

→ In collinear array, the antennas are arranged co-axially, that is antennas are mounted end to end in a single line.

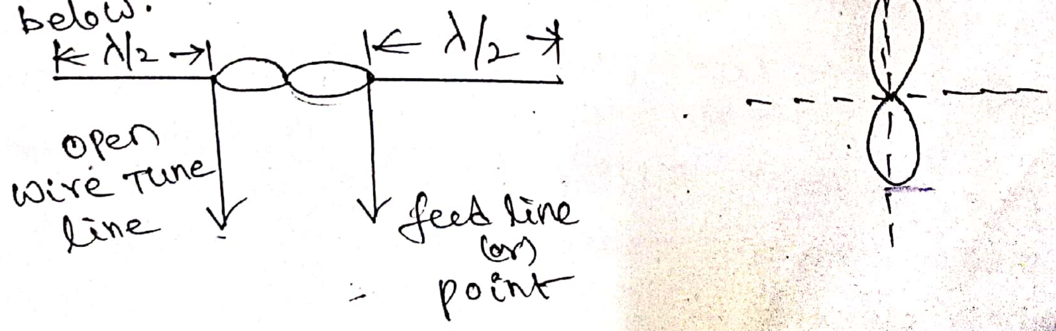
→ A collinear array is a broad side radiator, in which the direction of maximum radiation is perpendicular to line of antenna.





→ The power gain of collinear array is maximum when the spacing between elements is 0.3λ to 0.5λ.

→ The two element collinear array is given below.

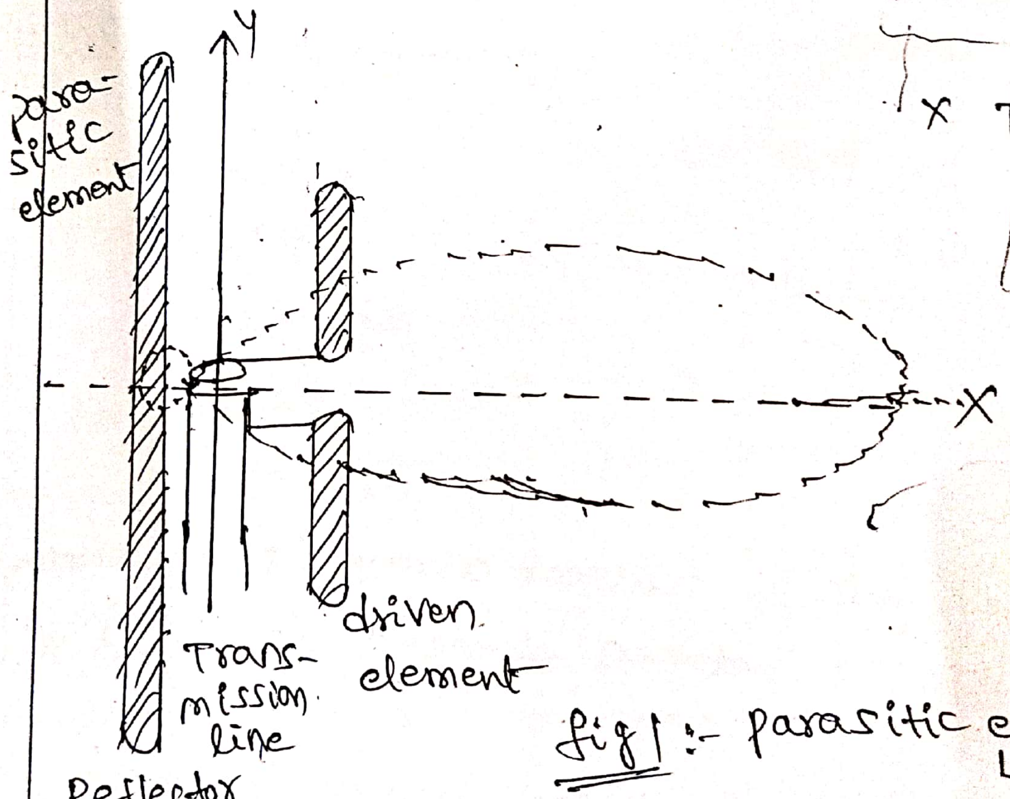


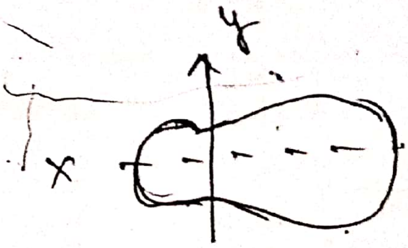
→ Two elements Collinear array is also Called the "Two half waves in phase".

4) parasitic arrays:-

→ Multielement arrays having number of parasitic elements are called "parasitic arrays". whether driven element is one (or) more.

→ a parasitic array with linear half wave dipole as elements is normally called as "yagi-Uda" (or) simply "yagi" antenna.





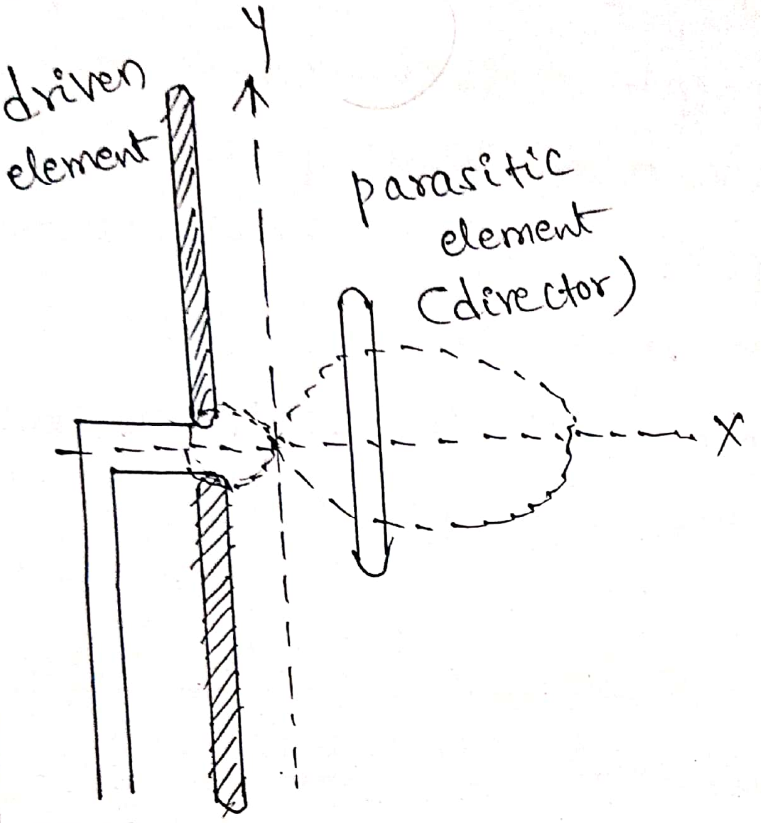
Radiation pattern.

fig1 :- parasitic elements Lengthend.

→ To overcome the feeding problems, it is desirable to use parasitic arrays.

→ The element supply power directly from source through transmission line called driven element

The amplitude and phase of the current ③ ~~element~~ induced in a parasitic element depends on tuning and spacing between parasitic element and driven element



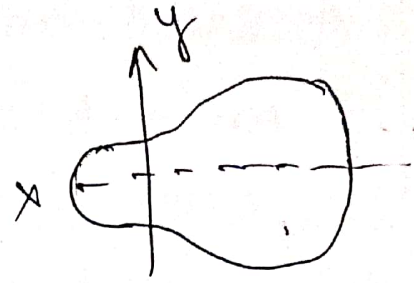


fig 2:- parasitic element shortend.

→ A parasitic element lengthend by 5%. with respect to driven element acts as reflector.

→ shortend by 5%. act as director.

TWO element arrays :- different cases :-

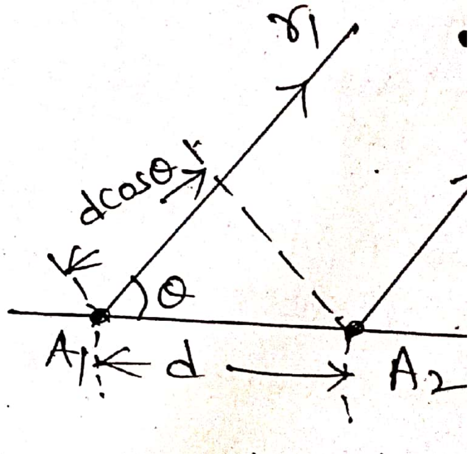
* The array of point sources nothing but the array of an isotropic radiators occupying zero volume.
* For the greater no. of point source in the array the analysis is very camplicated, and time Consuming.

* The simplest condition of no. of point sou
in the array is two.
There are 3 types of two element arrays

(i) Array of two point sources with curr
of Equal amplitude and same phase.
(ii) Array of two point sources with curr
of equal amplitude and opposite phase
(iii) Array of two point sources with curren
of unequal amplitude and any phase.

(i) Array of two point sources with curren
of equal amplitudes and same phase.

* consider two point sources A_1 and A_2 seperated by distance 'd'.
* Let both point sources are supplied with currents equal in Amplitude (or) magnitude, Same phase.



Two element array.

* The distance between point 'p' and A_1 is r_1 and distance between point 'p' and A_2 is r_2.

We can assume $r_1 = r_2 = r$.

$\therefore$ The path difference $= d\cos\theta$

In terms of wavelength, the path difference is

$$p.d = \frac{d\cos\theta}{\lambda} \rightarrow (1)$$

$\therefore$ The phase angle $\psi = 2\pi \times$ path difference

$$\Rightarrow \psi = 2\pi \times \frac{d\cos\theta}{\lambda}$$

$$\Rightarrow \psi = \frac{2\pi}{\lambda} \times d\cos\theta$$

$$\boxed{\psi = \beta d\cos\theta} \rightarrow (2)$$

Let E_1 be the far electric field at 'p' due to A_1.

$\Rightarrow$ E_2 be the far electric field at 'p' due to A_2.

The total field at point 'p' is given by

$$E = E_1 e^{-j\frac{\psi}{2}} + E_2 e^{j\psi/2}$$

assume equal amplitudes, same phase.

$$E_1 = E_2 = E_0$$

$$E = E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$E = E_0 (e^{-j\psi/2} + e^{j\psi/2}) \qquad (\because e^{j\theta} + e^{-j\theta} = 2\cos\theta)$$

$$E = E_0 \left(2\cos\frac{\psi}{2}\right)$$

$$E = E_0 \left(2\cos\left(\frac{\beta d\cos\theta}{2}\right)\right)$$

$$\therefore \boxed{E = 2E_0 \cos\left(\frac{\beta d\cos\theta}{2}\right)} \rightarrow (3)$$

Where E_0 = Max. amplitude.

$$\beta = \frac{2\pi}{\lambda}, \quad d = \lambda/2$$

~~Maxima direction~~ :-

The array factor is defined as the ratio magnitude of total field to magnitude of maxi[mum] field

$$A.F = \frac{|E|}{|\cancel{E}_{max}|} = \frac{|E|}{|2E_0|}$$

from eqn 3

$$A.F = \frac{E}{2E_0} = \cos\left(\frac{\beta d \cos\theta}{2}\right)$$

<u>maximum direction</u> :-

For maximum direction we have to equalize ± to Array factor.

$$\therefore \cos\left(\frac{\beta d \cos\theta}{2}\right) = \pm 1$$

$$\cos\left(\frac{\frac{2\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{2} \cos\theta}{2}\right) = \pm 1$$

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = \pm 1$$

$$\frac{\pi}{2}\cos\theta_{max} = \cos^{-1}(\pm 1)$$

$$\frac{\pi}{2}\cos\theta_{max} = \pm n\pi \quad \text{where } n = 0, 1, 2 \ldots$$

if $n = 0$ then

$$\frac{\pi}{2}\cos\theta_{max} = 0$$

$$\cos\theta_{max} = 0$$

$$\theta_{max} = \cos^{-1}(0)$$

$$\boxed{\theta_{max} = 90^\circ \text{ (or) } 270^\circ}$$

Minimum direction :-

The total field strength is minimum when $\cos\left(\frac{\pi}{2}\cos\theta\right)$ is '0'.

$$\left(\because \beta = \frac{2\pi}{\lambda},\ d = \frac{d}{2}\right)$$

$$\therefore \cos\left(\frac{\pi}{2}\cos\theta\right) = 0$$

$$\frac{\pi}{2}\cos\theta_{min} = \cos^{-1}(0)$$

$$\frac{\pi}{2}\cos\theta_{min} = \pm(2n+1)\frac{\pi}{2} \quad n = 0, 1, 2 \ldots$$

if $n=0$ then

$$\cancel{\frac{\pi}{2}}\cos\theta_{min} = \pm\cancel{\frac{\pi}{2}}$$

$$\theta_{min} = \cos^{-1}(\pm 1) \qquad \left(\because \cos 0^\circ = 1,\ \cos 180^\circ = -1\right)$$

$$\therefore \boxed{\theta_{min} = 0^\circ \text{ (or) } 180^\circ}$$

Half power point Direction :-

When the power is half the voltage (or) current is $\frac{1}{\sqrt{2}}$ times of maximum value.

$$\therefore \cos\left(\frac{\pi}{2}\cos\theta\right) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\theta_{HPPD} = \cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$$

$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm(2n+1)\frac{\pi}{4} \quad n = 0, 1, 2 \ldots$$

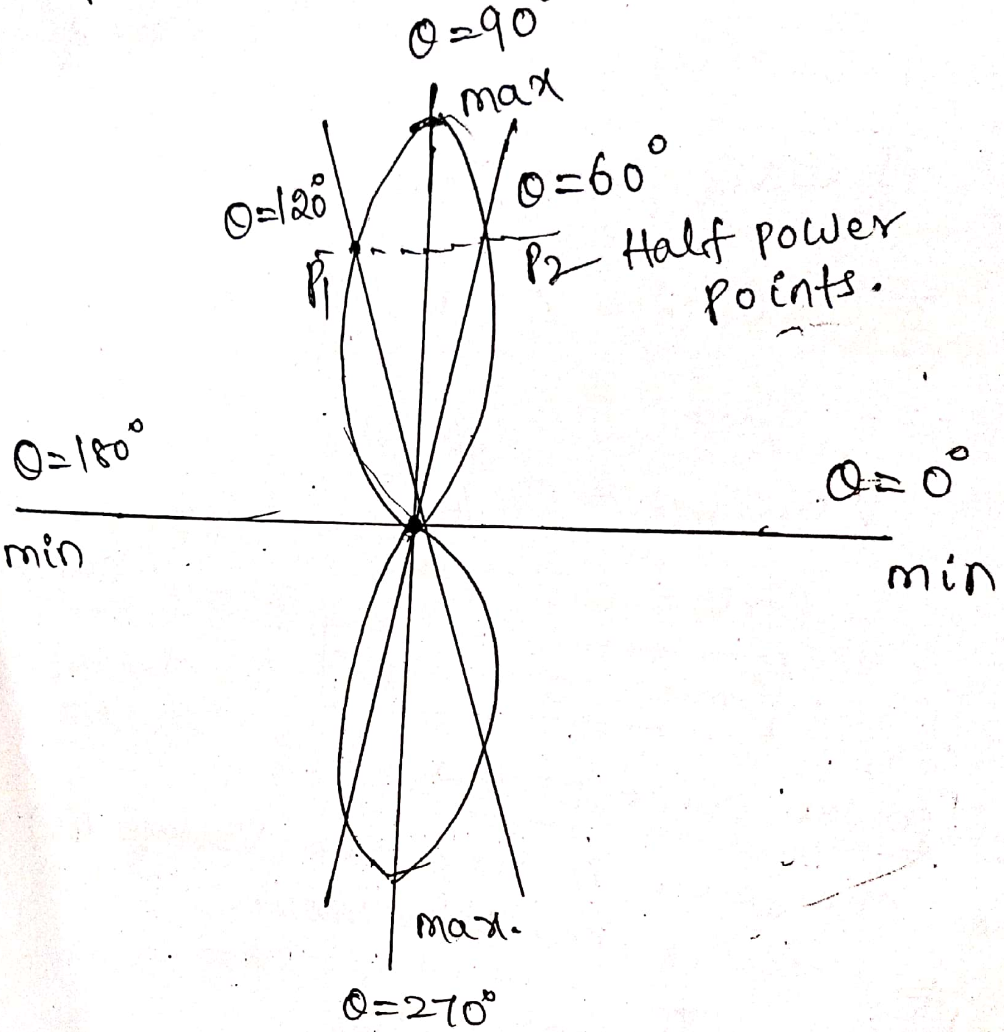
if $n=0$ then $\cancel{\frac{\pi}{2}}\cos\theta_{HPPD} = \pm\frac{\cancel{\pi}}{\cancel{4}\,2}$

$$\Rightarrow \cos\theta_{HPPD} = \pm\frac{1}{2} \qquad \left(\because \cos 60^\circ = \frac{1}{2},\ \cos 120^\circ = -\frac{1}{2}\right)$$

$$\Rightarrow \theta_{HPPD} = \cos^{-1}\left(\pm\frac{1}{2}\right)$$

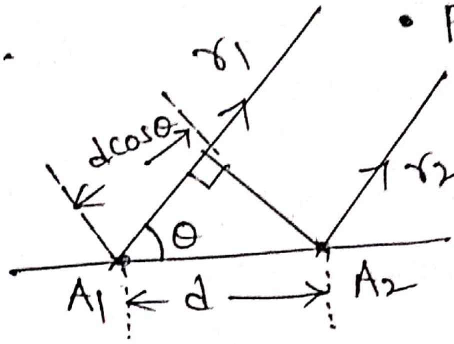
$$\therefore \boxed{\theta_{HPPD} = 60^\circ \text{ (or) } 120^\circ}$$

The field pattern for two element array with equal amplitudes and same phase is given below.



Arrays of two point sources with equal Amplitude and opposite phase:-

→ consider two point sources seperated by distance 'd' and supplied with currents equal in Amplitude but opposite phase.



'p' is distant point.

The total far field at distant point 'p' is given by

$$E = -E_1 e^{-j\psi/2} + E_2 e^{j\psi/2} \rightarrow (1)$$

Let $E_1 = E_2 = E_0$

The phase of source 1 is $-\frac{\psi}{2}$, phase of source 2 is $\frac{\psi}{2}$

$$\Rightarrow E = -E_0 e^{-j\frac{\psi}{2}} + E_0 e^{j\frac{\psi}{2}}$$

$$\Rightarrow E = E_0 \left[-e^{-j\frac{\psi}{2}} + e^{j\frac{\psi}{2}} \right]$$

$(\because e^{j\theta} - e^{-j\theta} = 2j\sin\theta)$

2cos
2j sine

$$\Rightarrow E = E_0 \left(2j \sin\frac{\psi}{2}\right)$$

$$\therefore \boxed{E = 2jE_0 \sin\left(\frac{\beta d\cos\theta}{2}\right)} \rightarrow (2)$$

$(\because \psi = \beta d \cdot \cos\theta)$

The array factor is given by

$$A.F = \frac{|E|}{|2jE_0|}$$

So eq (2) becomes

$$A.F = \frac{E}{2E_0} = \sin\left(\frac{\beta d\cos\theta}{2}\right)$$

$(\because \beta = \frac{2\pi}{\lambda}$, $d = \lambda/2)$

$$A.F = \frac{E}{2E_0} = \sin\left(\frac{\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos\theta}{2}\right)$$

$\therefore$ $\boxed{\text{array factor} = \sin\left(\frac{\pi}{2}\cos\theta\right)}$

Maximum direction :- The maximum value of sine function is ± 1

$$\sin\left(\frac{\pi}{2}\cos\theta\right) = \pm 1$$

$$\frac{\pi}{2}\cos\theta_{max} = \sin^{-1}(\pm 1)$$

where $n = 0, 1, 2 \ldots$

$$\Rightarrow \frac{\pi}{2}\cos\theta_{max} = \pm(2n+1)\frac{\pi}{2}$$

if $n=0$ then $\cancel{\frac{\pi}{2}}\cos\theta_{max} = \pm\cancel{\frac{\pi}{2}}$

$$\therefore \theta_{max} = \cos^{-1}(\pm 1)$$

$(\because \cos 0° = 1$
$\cos 180° = -1)$

$$\boxed{\theta_{max} = 0° \text{ (or) } 180°}$$

Minimum direction :- The minimum value of sine function is '0'

$$\sin\left(\frac{\pi}{2}\cos\theta\right) = 0$$

$$\frac{\pi}{2}\cos\theta_{min} = \sin^{-1}(0)$$

where

$$\frac{\pi}{2}\cos\theta_{min} = \pm n\pi \quad n = 0, 1, 2 \ldots$$

if $n=0$ then $\frac{\pi}{2}\cos\theta_{min} = 0$

$$\cos\theta_{min} = 0$$

$$\theta_{min} = \cos^{-1}(0)$$

$$\therefore \boxed{\theta_{min} = 90° \text{ (or) } 270°}$$

(7)

alf power point directions :-

$$\sin\left(\frac{\pi}{2}\cos\theta\right) = \pm\frac{1}{\sqrt{2}}$$

8

$$\frac{\pi}{2}\cos\theta_{HPPD} = \sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$$

$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm(2n+1)\frac{\pi}{4}$$ where $n=0,1,2\ldots$

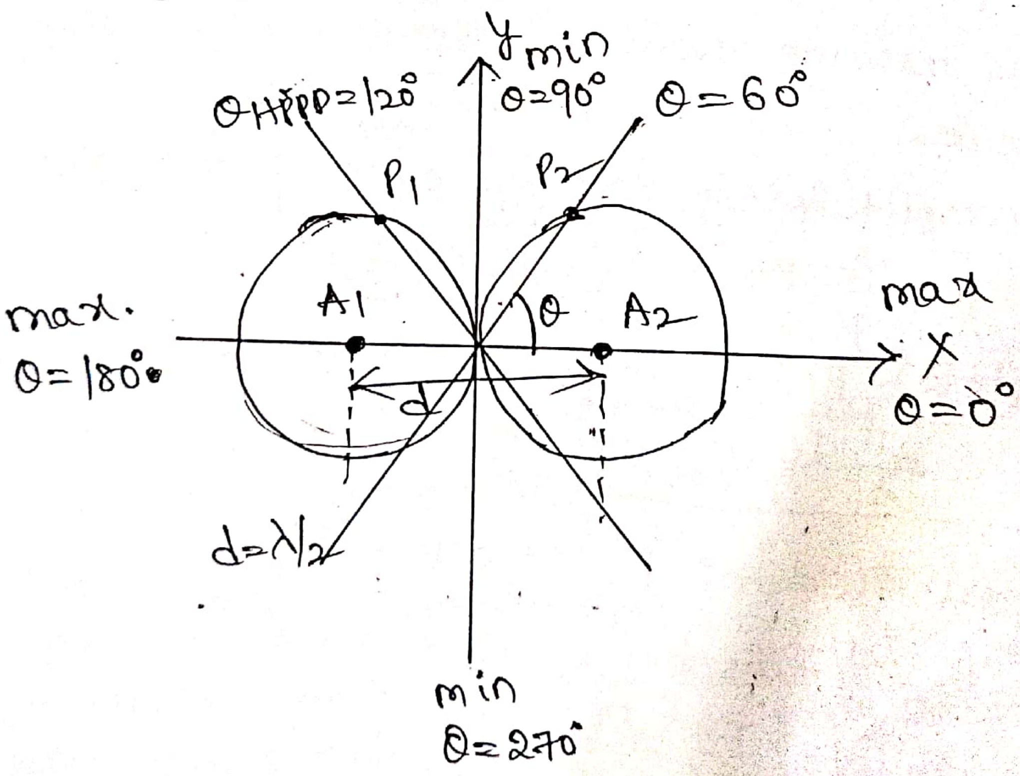
if $n=0$ then

$$\cancel{\frac{\pi}{2}}\cos\theta_{HPPD} = \pm\frac{\cancel{\pi}}{4/2}$$

$$\theta_{HPPD} = \cos^{-1}\left(\pm\frac{1}{2}\right)$$

$$\therefore \boxed{\theta_{HPPD} = 60^\circ \text{ and } 120^\circ}$$ ($\because \cos 60^\circ = \frac{1}{2}$, $\cos 120^\circ = -\frac{1}{2}$)

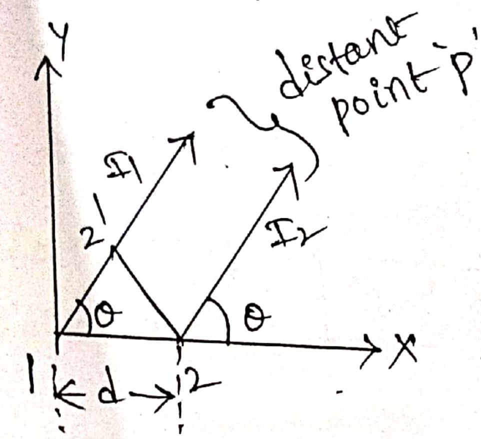
The field pattern for two point sources with spacing $d=\lambda/2$ and equal amplitudes, opposite phase (180°)



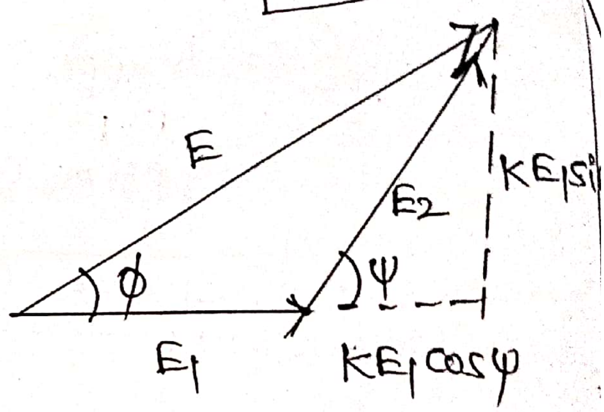
i) <u>Array of two point sources with unequal amplitudes and any phase.</u>

→ Let us now consider a general condition in which the amplitudes of two point sources are not equal and any phase difference say 'α'.

$$\boxed{E_2 = KE_1}$$



two point sources with unequal amplitudes & any phase



Vector diagram.

→ Let us assume source 1 is taken as reference for phase.

→ The amplitudes of source 1 and 2 at point 'P' are E_1 and E_2 ($\because E_1 > E_2$).

The total phase angle is given by

$$\boxed{\psi = \frac{2\pi}{\lambda} d\cos\theta + \alpha} \rightarrow (1)$$

The total field at 'P' is given by

$$E = E_1 e^{j.0} + E_2 e^{j\psi} = E_1 + E_2 e^{j\psi}$$

$$\Rightarrow E = E_1\left(1 + \frac{E_2}{E_1} e^{j\psi}\right)$$

$$\therefore \boxed{E = E_1(1 + K e^{j\psi})} \rightarrow (2)$$

where $K = \frac{E_2}{E_1}$

if $E_1 > E_2$ then $\frac{E_1}{E_2} > 1$

$\therefore \frac{E_2}{E_1} < 1$

$\therefore K < 1$

$$\boxed{0 \le K \le 1}$$

from equation (2) The magnitude and phase angle can be obtained.

$$E = \left| E_1 \{ 1 + K(\cos\psi + j\sin\psi) \} \right|$$

$$\therefore E = E_1 \sqrt{(1+K\cos\psi)^2 + (K\sin\psi)^2} \angle\phi$$

ϕ = phase angle at 'P'

$$\therefore \boxed{\phi = \tan^{-1}\left(\frac{K\sin\psi}{1+K\cos\psi}\right)}$$

principle of pattern multiplication :-

→ The pattern multiplication is a mathematical & simple method to obtain radiation patterns of arrays.

→ It is very useful in designing of arrays because it makes possible to draw the patterns of complicated arrays.

*. The total field pattern of an array of non-isotropic but similar sources is the multiplication of individual source patterns and pattern of an array of isotropic point sources each located at phase centre of individual source.

The total field pattern of an array of non isotropic but similar source is given by

$$E = \{E_i(\theta,\phi) \times E_a(\theta,\phi)\} \times \{E_{pi}(\theta,\phi) + E_{pa}(\theta,\phi)\}$$

where

E = Total field

$E_i(\theta,\phi)$ = field pattern of individual source

$E_a(\theta,\phi)$ = field pattern of array of isotropic so

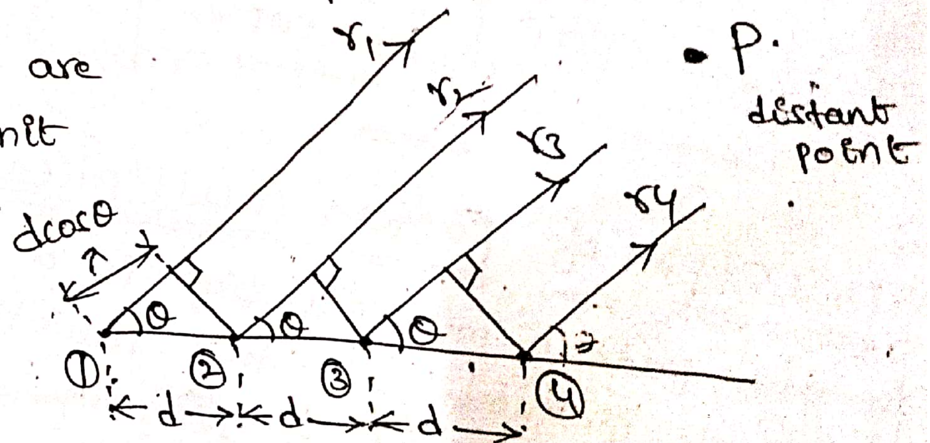
$E_{pi}(\theta,\phi)$ = phase pattern of individual source

$E_{pa}(\theta,\phi)$ = phase pattern of array of isotropic point source.

Radiation pattern of 4-Isotropic elements fed in phase spaced $\frac{\lambda}{2}$ apart:-

→ Two isotropic point sources spaced $\frac{\lambda}{2}$ apart fed in phase provides a bidirectional pattern.

→ Elements ① and ② are considered as one unit and is to be placed between the middle of the elements.



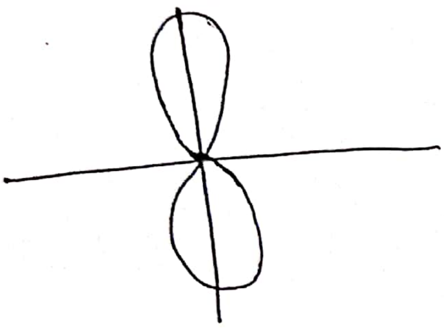
→ Also the elements ③ ④ are considered as one unit assumed to be placed between the middle of the two elements.

Fig:-a four element linear array.

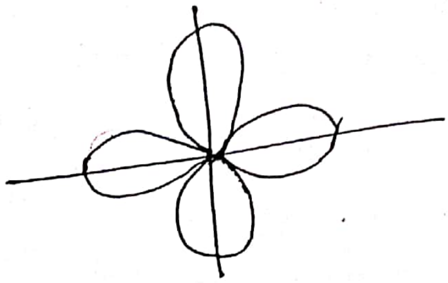
Now we can replace elements ① and ② by a single antenna located at a point midway between them $(\frac{d}{2})$.

Similarly replace elements ③ and ④ by single antenna having same pattern.

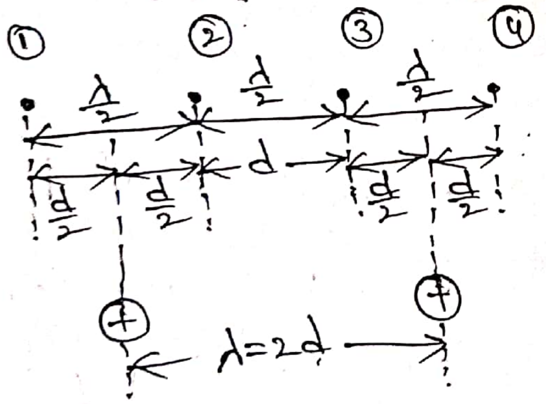
The resultant radiation pattern of four elements array can be obtained as multiplication of patterns.



(a) radiation pattern of two antennas spaced at distance $\frac{\lambda}{2}$ and with equal currents, same phase.

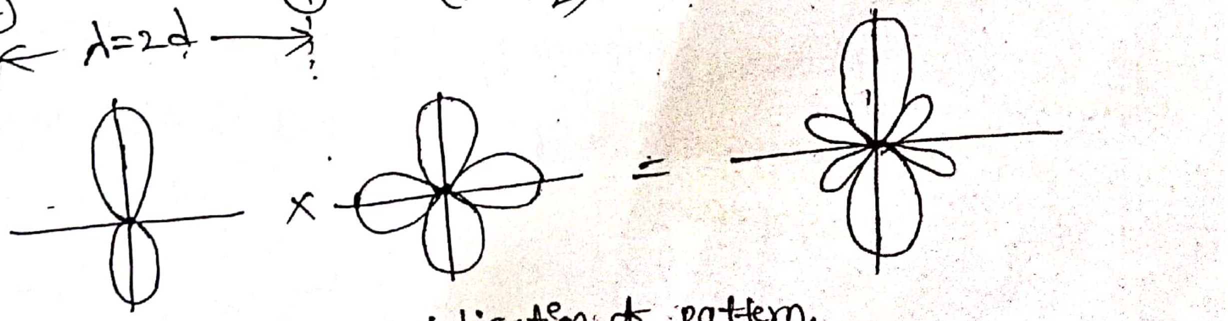


(b) radiation pattern of two antennas spaced at distance 'λ' with equal currents, same phase.



$(\because d = \frac{\lambda}{2})$

(c) Antennas ①, ② and ③, ④ replaced by single antenna seperately



(d) multiplication of pattern.

n element uniform linear array:-

→ An array is said to be linear, if the individual elements of the array are spaced equally along a line and it is uniform if the array are fed with currents of equal amplitude & uniform progressive phase shift.

→ Now we shall calculate pattern of linear array of 'n' isotropic point sources which are spaced equally.

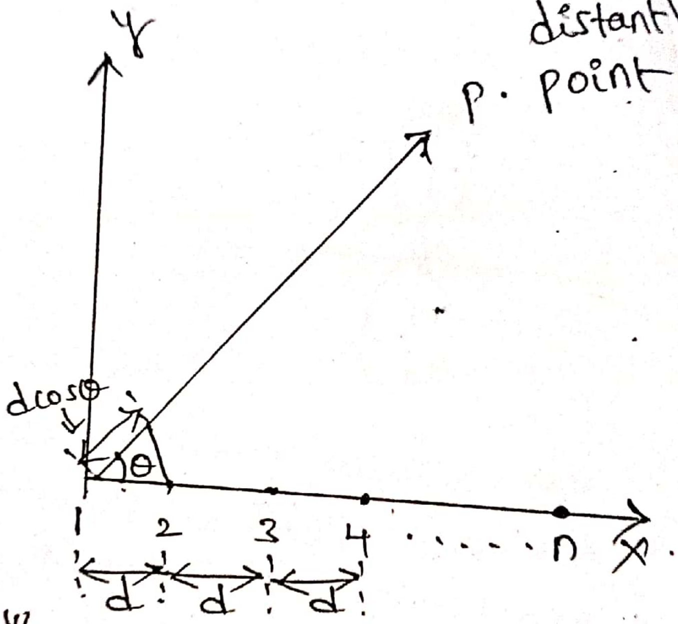


fig:- Linear array with 'n' isotropic point sources.

→ The total far field at distant point 'P' is

$$E_t = E_0 e^{j\cdot 0} + E_0 e^{j\psi} + E_0 e^{j2\psi} + E_0 \cdot e^{j3\psi} + \dots + E_0 e^{j(n-1)\psi}$$

⇒ $$\boxed{E_t = E_0 \left[1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + e^{j4\psi} + \dots e^{j(n-1)\psi}\right]} \rightarrow (1)$$

where $\psi = (\beta d \cos\theta + \alpha)$ radian

= Total phase difference of fields at 'P'

α = phase difference in adjacent sources (or) progressive phase shift b/w two point sources.

Multiplying eq ① by $e^{j\psi}$

$$E_t e^{j\psi} = E_0\left(e^{j\psi} + e^{2j\psi} + e^{j3\psi} + e^{j4\psi} + \dots + e^{jn\psi}\right) \rightarrow ②$$

Subtracting eq ② from ①

$$① - ② \Rightarrow E_t - E_t e^{j\psi} = \left\{E_0\left(1 + \cancel{e^{j\psi}} + \cancel{e^{2j\psi}} + \cancel{e^{j3\psi}} + \dots + \cancel{e^{j(n-1)\psi}}\right\}\right.$$

$$-\left\{E_0\left(\cancel{e^{j\psi}} + \cancel{e^{j2\psi}} + \cancel{e^{j3\psi}} + \dots + e^{jn\psi}\right\}\right.$$

$$\therefore E_t(1 - e^{j\psi}) = E_0(1 - e^{jn\psi})$$

$$\Rightarrow E_t = E_0\left[\frac{1 - e^{jn\psi}}{1 - e^{j\psi}}\right] = E_0\left[\frac{1 - e^{jn\frac{\psi}{2}} \cdot e^{jn\frac{\psi}{2}}}{1 - e^{j\frac{\psi}{2}} \cdot e^{j\frac{\psi}{2}}}\right] \rightarrow ③$$

$$= E_0\left[\frac{e^{jn\frac{\psi}{2}} \cdot e^{-jn\frac{\psi}{2}} - e^{jn\frac{\psi}{2}} \cdot e^{jn\frac{\psi}{2}}}{e^{j\frac{\psi}{2}} \cdot e^{-j\frac{\psi}{2}} - e^{j\frac{\psi}{2}} \cdot e^{j\frac{\psi}{2}}}\right]$$

$$= E_0\left[\frac{e^{jn\frac{\psi}{2}}\left(e^{-jn\frac{\psi}{2}} - e^{jn\frac{\psi}{2}}\right)}{e^{j\frac{\psi}{2}}\left(e^{-j\frac{\psi}{2}} - e^{j\frac{\psi}{2}}\right)}\right]$$

$$= E_0\left[\frac{-\cancel{2j} \times \sin n\frac{\psi}{2}}{-\cancel{2j} \times \sin\frac{\psi}{2}}\right] \cdot e^{j(n-1)\frac{\psi}{2}}$$

$$\therefore E_t = E_0\left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right] e^{j\phi}$$

Where $\phi = \frac{(n-1)}{2}\psi \rightarrow ④$

$$|E_t| = \left|E_0\left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right]\left[\cos\phi + j\sin\phi\right]\right|$$

$$\therefore \boxed{E_t = E_0 \left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right] \angle\phi}$$ Where $\phi = \left(\frac{n-1}{2}\right)\psi$

The total far field pattern of Linear array of n- isotropic point source is

$$\boxed{E_t = E_0 \left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right]}$$

There are 3 different cases under the n-element uniform linear arrays.

(1). broad side array

(2) End fire array

(3) End fire array with increased directivity

broad side array :-

→ An array is said to be broad side array, if the maximum direction of radiation perpendicular to the line of array (ie) 90° and 270°. Broad side sources are in phase. $\alpha = 0°$, $\psi = 0$ for maximum.

$$\therefore \psi = \beta d \cos\theta + \alpha = \beta d \cos\theta + 0$$

$$\Rightarrow 0 = \beta d \cos\theta$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0) = 90° \text{ (or) } 270°$$

$$\therefore \boxed{\theta = 90° \text{ (or) } 270°}$$

Directions of pattern ~~[illegible]~~ maxima:- (minor lobe) (11)

For array of n-isotropic point sources of equal amplitude & spacing we are using S.A Schelkunoff procedure.

$$E_t = E_0 \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}$$

this is maximum when numerator is maximum

$\therefore \sin n\frac{\psi}{2} = 1$

$n\frac{\psi}{2} = \sin^{-1}(1) = \pm(2N+1)\frac{\pi}{2}$

$N = 1, 2, 3, 4 \ldots$

$N = 0$ for major lobe maxima.

$\Rightarrow \frac{\psi}{2} = \pm(2N+1)\frac{\pi}{2} \times \frac{1}{n}$

$\psi = \pm\frac{(2N+1)\pi}{n}$

$\beta d\cos(\theta_{max})_{minor} + \alpha = \pm\frac{(2N+1)\pi}{n}$

$\beta d\cos(\theta_{max})_{minor} = \pm\frac{(2N+1)\pi}{n} - \alpha$

$\cos(\theta_{max})_{minor} = \frac{1}{\beta d}\left[\pm\frac{(2N+1)\pi}{n} - \alpha\right]$

$$\boxed{(\theta_{max})_{minor} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{(2N+1)\pi}{n} - \alpha\right\}\right]}$$

where $(\theta_{max})_{minor}$ = minor lobe maxima

For broad side array $\alpha = 0$.

$(\theta_{max})_{minor} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{(2N+1)\pi}{n}\right\}\right]$

$$\Rightarrow (\theta_{max})_{minor} = \cos^{-1}\left[\frac{1}{\frac{2\pi}{\lambda}\cdot d}\left\{\pm\frac{(2N+1)\cancel{\pi}}{n}\right\}\right]$$

$$\boxed{\therefore (\theta_{max})_{minor} = \cos^{-1}\left[\pm\frac{(2N+1)\lambda}{2nd}\right]}$$

For example

Let $n=4,\ d=\lambda/2 \ :\ \alpha=0$

$$(\theta_{max})_{minor} = \cos^{-1}\left[\frac{(2N+1)\cancel{\lambda}}{\cancel{2}\cdot 4\cdot\frac{\cancel{\lambda}}{\cancel{2}}}\right] = \cos^{-1}\left(\pm\frac{(2N+1)}{4}\right)$$

for $N=1$ $(\theta_{max})_{minor} = \cos^{-1}(\pm\frac{3}{4})$

$= \pm 0.7\ldots$ radians

$= \pm 41.4°$ degrees.. ($\because$ 1rad $= 57.3°$)

(or)

$= \pm 138.6°$ degrees.

$$\therefore \boxed{(\theta_{max})_{minor} = \pm 41.4° \text{ (or) } \pm 138.6°}$$

Direction of pattern minima:-

According to S.A schelkunoff procedure

$$E_t = E_0 \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}$$

$$\Rightarrow \sin n\frac{\psi}{2} = 0 \qquad (\because \sin\frac{\psi}{2} \neq 0)$$

$$n\frac{\psi}{2} = \sin^{-1}(0) = \pm N\pi, \quad N = 1,2,3,\ldots$$

$$\psi = \pm\frac{2N\pi}{n}$$

$$\beta d\cos(\theta_{min})_{minor} \mp \alpha = \pm\frac{2N\pi}{n}$$

$$\cos(\theta_{min})_{minor} = \frac{1}{\beta d}\left\{\pm\frac{2N\pi}{n} - \alpha\right\}$$

$$\boxed{(\theta_{min})_{minor} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{2N\pi}{n} - \alpha\right\}\right]}$$

For broad side $\alpha = 0$, $\beta = \frac{2\pi}{\lambda}$

$$\therefore (\theta_{min})_{minor} = \cos^{-1}\left[\frac{1}{\frac{2\pi}{\lambda}\cdot d}\left(\pm\frac{2N\pi}{n}\right)\right]$$

$$= \cos^{-1}\left[\pm\frac{2N\lambda}{2nd}\right]$$

$$\boxed{\therefore (\theta_{min})_{minor} = \cos^{-1}\left[\pm\frac{N\lambda}{nd}\right]}$$

For example if $n=4$, $d = \lambda/2$

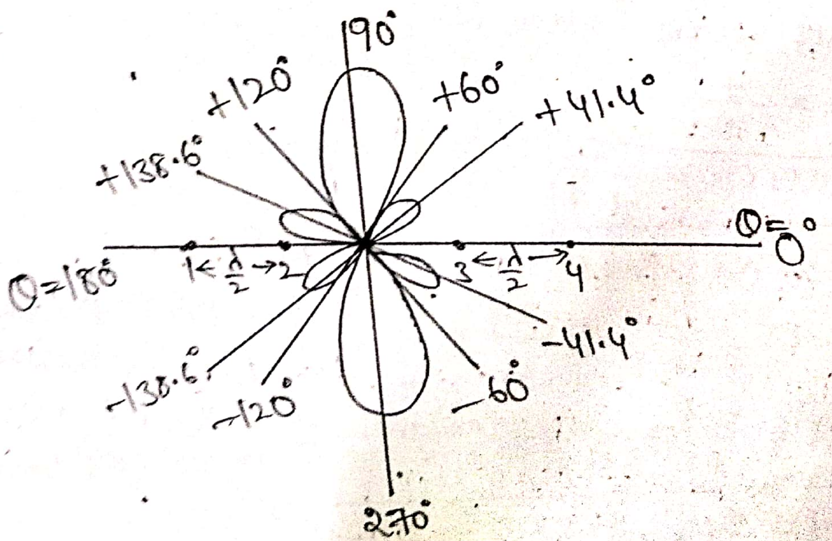
$$(\theta_{min})_{minor} = \cos^{-1}\left[\pm\frac{1\cdot\lambda}{4\cdot\frac{\lambda}{2}}\right] = \cos^{-1}\left(\pm\frac{1}{2}\right) \quad \text{for } N=1$$

$$\boxed{\therefore (\theta_{min})_{minor} = \pm 60^\circ, \pm 120^\circ}$$

for $N=2$

$$(\theta_{min})_{minor} = \cos^{-1}\left[\pm\frac{2\cdot\lambda}{4\cdot\frac{\lambda}{2}}\right] = \cos^{-1}(\pm 1)$$

$$\boxed{\therefore (\theta_{min})_{minor} = 0^\circ, 180^\circ.}$$



Beam width of major lobe:-

It is defined as
(i) The angle between first nulls (or)
(ii) The double angle between first null and major lobe maximum directions.

Let the Complementary angle, $\gamma = 90° - \theta_{min}$.

$$\Rightarrow \theta_{min} = 90° - \gamma$$

Beam width of major lobe = 2x angle between first null and major lobe maximum.

$$\Rightarrow BWFN = 2\times\gamma$$

But $\theta_{min} = \cos^{-1}\left(\pm \frac{N\lambda}{nd}\right)$

$$90° - \gamma = \cos^{-1}\left(\pm\frac{N\lambda}{nd}\right)$$

$$\cos(90° - \gamma) = \pm\frac{N\lambda}{nd}$$

$$\sin\gamma = \pm\frac{N\lambda}{nd} \qquad (\because \gamma \text{ is very small } \sin\gamma \simeq \gamma)$$

$$\boxed{\gamma = \pm\frac{N\lambda}{nd}}$$

first null occurs when $N = 1$

$$\gamma = \frac{\pm\lambda}{nd}$$

$$\therefore \boxed{BWFN = \frac{2\lambda}{nd}}$$

if $N\lambda >> nd$ then

$$2\gamma = \frac{2\lambda}{nd} = \frac{2\lambda}{L} \qquad (\because L = (n-1)d = \text{Total Length of array})$$

$$\therefore 2\gamma = \frac{2\lambda}{L} = \frac{2}{(L/\lambda)} \text{ radians. } L \simeq nd. \text{ (if } n \text{ is very large)}$$

$$\boxed{BWFN = 2\gamma = \frac{2\times 57.3°}{(L/\lambda)} = \frac{114.6°}{(L/\lambda)}}$$

the half power beam width is

$$HPBW = \frac{BWFN}{2} = \frac{1}{(L/\lambda)} \text{ radians.}$$

$$\boxed{HPBW = \frac{57.3}{(L/\lambda)} \text{ degrees}}$$

End fire array :-

→ An array is said to be end fire, if the maximum direction of radiation coincides with the array axis (or) line (ie) $\theta = 0°$ (or) $180°$.

$\therefore \psi = 0$, and $\theta = 0°$ (or) $180°$

$$\psi = \beta d \cos\theta + \alpha$$

$$0 = \beta d \cos 0° + \alpha$$

$$0 = \beta d + \alpha$$

$$\boxed{\alpha = -\beta d}$$

$\left(\because \beta = \frac{2\pi}{\lambda},\ d = \frac{\lambda}{2}\right.$

$\left.\alpha = -\frac{\cancel{2}\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{\cancel{2}} = -180°\right)$

direction of pattern maxima :-

According to S.A schelkunoff procedure

$$\sin n\frac{\psi}{2} = 1$$

$$n\frac{\psi}{\cancel{2}} = \sin^{-1}(1) = \pm(2N+1)\frac{\pi}{\cancel{2}}$$

$$\Rightarrow n\psi = \pm(2N+1)\pi$$

$$\psi = \pm\frac{(2N+1)\pi}{n}$$

$$\beta d \cos(\theta_{max})_{minor} + \alpha = \pm\frac{(2N+1)\pi}{n}$$

$$\beta d \cos(\theta_{max}) - \beta d = \pm\frac{(2N+1)\pi}{n}$$

$\rightarrow \beta d\left(\cos(\theta_{max})_{minor} - 1\right) = \pm \frac{(2N+1)\pi}{n}$

$$\cos(\theta_{max})_{minor} - 1 = \pm \frac{(2N+1)\pi}{\beta n d}$$

$$\cos(\theta_{max})_{minor} = \pm \frac{(2N+1)\pi}{\beta n d} + 1$$

$$\boxed{(\theta_{max})_{minor} = \cos^{-1}\left[\pm \frac{(2N+1)\pi}{\beta n d} + 1\right]}$$

if $n = 4$, $d = \lambda/2$, $\alpha = -\pi$

for $N = 1$

$$(\theta_{max})_{minor} = \cos^{-1}\left[\pm \frac{(2\cdot 1+1)\cancel{\pi}}{\frac{2\cancel{\pi}}{\cancel{\lambda}}\cdot 4 \cdot \frac{\cancel{\lambda}}{2}} + 1\right]$$

$$= \cos^{-1}\left[\pm \frac{3}{4} + 1\right] = \cos^{-1}\left[\frac{7}{4}, \frac{1}{4}\right]$$

But $\cos^{-1}\left(\frac{7}{4}\right)$ doesn't exist

$\therefore (\theta_{max})_{minor} = \cos^{-1}\left(\frac{1}{4}\right) = 75.5^\circ$

$$\boxed{\therefore (\theta_{max})_{minor} = 75.5^\circ}$$

for $N = 2$

$$(\theta_{max})_{2\,minor} = \cos^{-1}\left[\pm \frac{(2\cdot 2+1)\cancel{\pi}}{\frac{2\cancel{\pi}}{\cancel{\lambda}}\cdot 4 \cdot \frac{\cancel{\lambda}}{2}} + 1\right] = \cos^{-1}\left(\pm \frac{5}{4} + 1\right)$$

$$= \cos^{-1}\left(\frac{5}{4} + 1, -\frac{5}{4} + 1\right)$$

$$= \cos^{-1}\left(\frac{-1}{4}\right) = -75.5^\circ$$

$$\boxed{(\theta_{max})_{2}\,minor = -75.5^\circ}$$

Directions of pattern minima:-

According to S.A. Schelkunoff procedure

$$\sin \frac{n\psi}{2} = 0$$

$$\frac{n\psi}{2} = \sin^{-1}(0) = \pm N\pi \qquad N = 1, 2, 3 \ldots$$

$$\psi = \pm \frac{2N\pi}{n}$$

$$\beta d \cos(\theta_{min})_{minor} + \alpha = \pm \frac{2N\pi}{n}$$

$$\beta d \cos(\theta_{min})_{minor} - \beta d = \pm \frac{2N\pi}{n} \qquad (\because \alpha = -\beta d)$$

$$\beta d\{(\cos\theta_{min})_{minor} - 1\} = \pm \frac{2N\pi}{n}$$

$$\cos(\theta_{min})_{minor} - 1 = \pm \frac{2N\pi}{\beta n d} = \frac{\pm \cancel{2}N\cancel{\pi}}{\frac{\cancel{2\pi}}{\lambda} \cdot n \cdot d}$$

$$\therefore (\cos\theta_{min} - 1) = \pm \frac{N\lambda}{nd}$$

$$\left(\cos 2 \cdot \frac{\theta_{min}}{2} - 1\right) = \pm \frac{N\lambda}{nd}$$

$$\cancel{1} - 2\sin^2 \frac{\theta_{min}}{2} - \cancel{1} = \pm \frac{N\lambda}{nd}$$

$$2\sin^2 \frac{\theta_{min}}{2} = \pm \frac{N\lambda}{nd}$$

$$\sin^2 \frac{\theta_{min}}{2} = \pm \frac{N\lambda}{2nd}$$

$$\sin \frac{\theta_{min}}{2} = \pm\sqrt{\frac{N\lambda}{2nd}}$$

$$\frac{\theta_{min}}{2} = \sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right)$$

$$\boxed{(\theta_{min})_{minor} = 2\sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right)}$$

For example $n=4,\ d=\frac{\lambda}{2}$

$$N=1 \quad (\theta_{min})_1 = 2\sin^{-1}\left(\pm\sqrt{\frac{1\cdot\cancel{\lambda}}{\cancel{2}\cdot 4\cdot\frac{\cancel{\lambda}}{\cancel{2}}}}\right)$$

$$= 2\sin^{-1}\left(\pm\frac{1}{2}\right) = 2\times(\pm 30) = \pm 60^\circ$$

$$N=2 \quad (\theta_{min})_2 = 2\sin^{-1}\left(\pm\sqrt{\frac{\cancel{2}\cdot\cancel{\lambda}}{\cancel{2}\cdot 4\cdot\frac{\cancel{\lambda}}{2}}}\right) = 2\sin^{-1}\left(\pm\sqrt{\frac{1}{2}}\right)$$

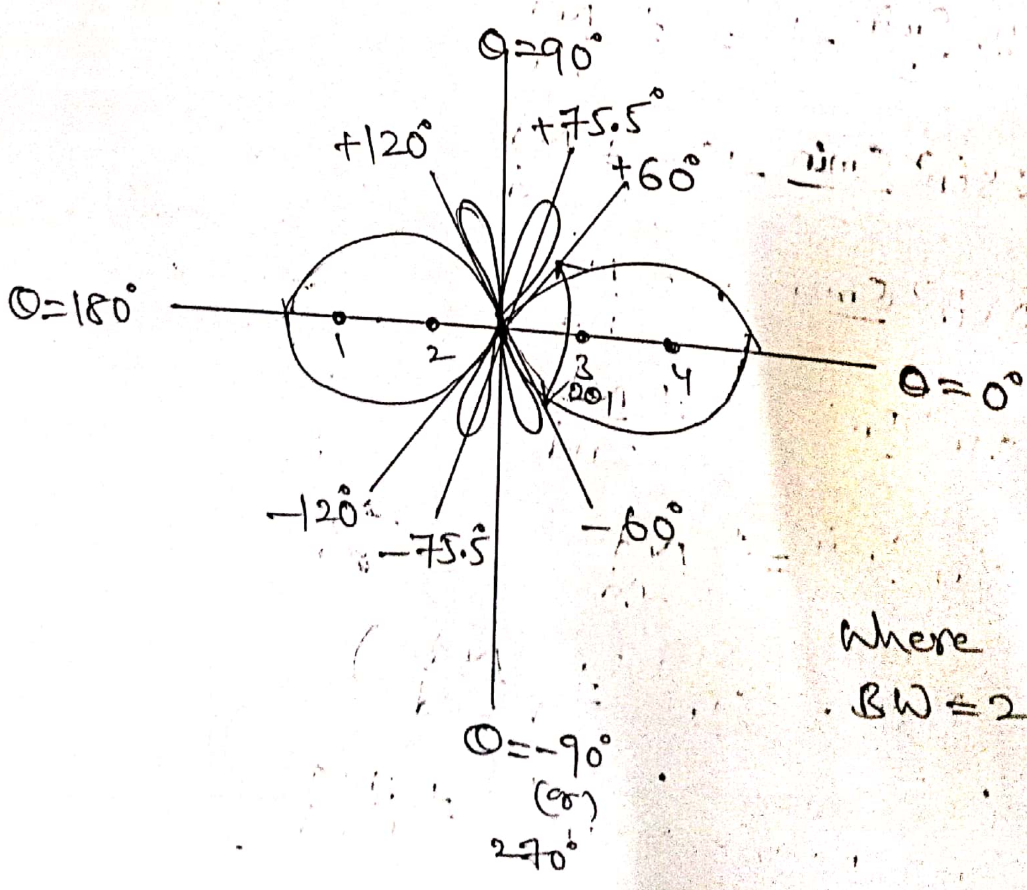
$$= 2\sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right) = 2\times(\pm 45^\circ) = \pm 90^\circ$$

$$N=3 \quad (\theta_{min})_3 = 2\sin^{-1}\left(\pm\sqrt{\frac{3\cdot\cancel{\lambda}}{\cancel{2}\cdot 4\cdot\frac{\cancel{\lambda}}{\cancel{2}}}}\right) = 2\sin^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$$

$$= 2\times(\pm 60^\circ) = \pm 120^\circ$$

$$N=4 \quad (\theta_{min})_4 = 2\sin^{-1}\left(\pm\sqrt{\frac{\cancel{4}\cdot\cancel{\lambda}}{\cancel{2}\cdot\cancel{4}\cdot\frac{\cancel{\lambda}}{2}}}\right) = 2\sin^{-1}(\pm 1)$$

$$= 2\times(\pm 90^\circ) = \pm 180^\circ.$$



Where

$BW = 2\theta_1.$

...am width of major lobes:-

The complementary angle r is not required in this End fire array case. because the beam width of end fire array is larger than broad side.

$\therefore$ Beam width $= 2\times$ angle between first nulls & maximum of major lobe

$$\boxed{BW = 2\times\theta_1}$$

$$\theta_{min} = 2\sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right)$$

$$\sin\left(\frac{\theta_{min}}{2}\right) = \pm\sqrt{\frac{N\lambda}{2nd}}$$

$$\Rightarrow \sin\left(\frac{\theta_{min}}{2}\right) \cong \frac{\theta_{min}}{2} = \pm\sqrt{\frac{N\lambda}{2nd}}$$

$$\therefore \boxed{\theta_{min} = \pm 2\sqrt{\frac{N\lambda}{2nd}} = \pm\sqrt{\frac{2N\lambda}{nd}}}$$

if L is Total length of array

$$L = (n-1)d \Rightarrow \boxed{L \cong nd}$$

$$\boxed{\theta_{min} = \pm\sqrt{\frac{2N\lambda}{nd}} = \pm\sqrt{\frac{2N\lambda}{L}}}$$

Beam width between first nulls (BWFN) $= 2\times\theta_{min}$

$$\therefore BWFN = 2\times\left(\pm\sqrt{\frac{2N\lambda}{nd}}\right) = \pm 2\sqrt{\frac{2N\lambda}{nd}}$$

$$\Rightarrow 2\theta_1 = \pm 2\sqrt{\frac{2N}{(L/\lambda)}} = \pm 2\sqrt{\frac{2\times 1}{(L/\lambda)}} \text{ for } N=1$$

$$\therefore BWFN = \pm 2\sqrt{\frac{2}{\left(\frac{L}{\lambda}\right)}} \text{ rad} = \pm 2\times 57.3^\circ\sqrt{\frac{2}{(L/\lambda)}} \text{ degree}$$

$$\boxed{BWFN = \pm 114.6\sqrt{\frac{2}{(L/\lambda)}}} \qquad \boxed{HPBW = \frac{BWFN}{2} = \pm 57.3\sqrt{\frac{2}{\left(\frac{L}{\lambda}\right)}}}$$

③ End fire array with increased directivity:-

→ The maximum radiation can be obtained along the axis of the uniform array by allowing progressive phase shift 'α' between elements equal to $\pm \beta d$.

$\alpha = -\beta d$ for $\theta = 0°$, $\quad$ ($\because \psi = \beta d \cos\theta + \alpha$)

$\alpha = +\beta d$ for $\theta = 180°$ $\quad$ and $\psi = 0$ for max.

$\Rightarrow 0 = \beta d \cos\theta + \alpha$

$\alpha = -\beta d \cos\theta$)

→ This produces a maximum field in the direction $\theta = 0°$ but does not give maximum directivity.

→ To improve the directivity of an end fire array without destroying other characteristics:

→ In 1938 Hansen and Woodyard proposed, the required phase shift between closely spaced elements of a very long array should be

$$\alpha = -\left(\beta d + \frac{\pi}{n}\right) \cong -\left(\beta d + \frac{2.94}{n}\right) \text{ for maximum } \theta = 0. \rightarrow (1)$$

$$\alpha = +\left(\beta d + \frac{\pi}{n}\right) \cong +\left(\beta d + \frac{2.94}{n}\right) \text{ for maximum } \theta = 180° \rightarrow (2)$$

These conditions are referred to as "Hansen Woodyard conditions for Increased directivity".

→ The above conditions also cannot achieve maximum directivity at $\theta = 0°$ and $\theta = 180°$

The magnitude of maximum value is not be unity and side lobe level is not −13.46 db.

→ To increase the directivity due to "Hansen-Woodyard conditions from eqns ①,② with assumptions of $|\psi|$ values.

(i) For maximum radiation along $\theta = 0$:-

$$|\psi| = |\beta d \cos\theta + \alpha|_{\theta=0} \cong \frac{\pi}{n} \rightarrow (3)$$

$$|\psi| = |\beta d \cos\theta + \alpha|_{\theta=180^\circ} \cong \pi \rightarrow (4)$$

(ii) For maximum radiation along $\theta = 180^\circ$

$$|\psi| = |\beta d \cos\theta + \alpha|_{\theta=180^\circ} \cong \frac{\pi}{n} \rightarrow (5)$$

$$|\psi| = |\beta d \cos\theta + \alpha|_{\theta=0^\circ} \cong \pi \rightarrow (6)$$

→ The main requirement is to fulfil the condition $|\psi| = \pi$ for each array

→ For array of n-elements the condition $|\psi| = \pi$ is satisfied by using eqns ①,② for $\theta = 0^\circ$ and $\theta = 180^\circ$

The spacing between two elements is

$$d = \left(\frac{n-1}{n}\right)\frac{\lambda}{4}$$

if the no. of elements considered is large then

$$d = \frac{\cancel{n}}{\cancel{n}} \cdot \frac{\lambda}{4} \Rightarrow \boxed{d = \frac{\lambda}{4}}$$

→ Hence for large uniform array the spacing is $\frac{\lambda}{4}$ to increase the directivity.

Comparision of characteristics :-

S.No	Type of array	Directions of minor lobe
1.	General case	$(\theta_{max})_{minor} = \cos^{-1}\left[\pm \frac{(2N+1)\pi}{n} - \alpha\right.$
2.	Broad side $(\alpha = 0)$	$(\theta_{max})_{minor} = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm \frac{(2N+1)\pi}{n}\right]\right.$
3.	ordinary End fire ($\cancel{\alpha = \pm\beta d}$) $\alpha = \pm \beta d$	$(\theta_{max})_{minor} = \cos^{-1}\left[\pm \frac{(2N+1)\pi}{\beta n d} + 1\right]$ $= \cos^{-1}\left[\pm \frac{(2N+1)\lambda}{2nd} + 1\right]$

S.No	Type of array	Directions of minor lobe minima
1.	General case	$(\theta_{min})_{minor} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm \frac{2N\pi}{n} - \alpha\right.\right.$
2.	Broad side $(\alpha = 0)$	$(\theta_{min})_{minor} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm \frac{2N\pi}{n}\right\}\right]$ $= \cos^{-1}\left[\pm \frac{N\lambda}{nd}\right]$
3.	ordinary End fire $\alpha = \pm \beta d$	$(\theta_{min})_{minor} = 2\sin^{-1}\left(\pm \sqrt{\frac{N\lambda}{2nd}}\right)$

S.No	Type of array	Beam width between first nulls
1.	Broad side array	$BWFN = \frac{2\lambda}{nd} = \frac{114.6^\circ}{(L/\lambda)}$
2.	ordinary End fire array	$BWFN = 2\sqrt{\frac{2N\lambda}{nd}}$ rad $= 114.6^\circ \sqrt{\frac{2}{(L/\lambda)}}$

Type of array	HPBW (Half power Beam width)
Broad side array	$HPBW = \frac{57.3^\circ}{\left(\frac{L}{\lambda}\right)} = \frac{\lambda}{nd}$ rad
End fire array	$HPBW = 57.3^\circ \sqrt{\frac{2}{(L/\lambda)}} = \sqrt{\frac{2}{\left(\frac{L}{\lambda}\right)}}$ rad $(\because 1\,rad = 57.3^\circ)$
End fire array with Increased directivity.	$HPBW = \frac{52^\circ}{\sqrt{L/\lambda}}$

Directivity relations:-

For a broad side array

$$\boxed{D = 2n\left(\frac{d}{\lambda}\right)}$$

$$\Rightarrow D = 2\left(\frac{nd}{\lambda}\right)$$

$$\Rightarrow \boxed{D = 2\left(\frac{L}{\lambda}\right)}$$

$L = (n-1)d$. if n is large

$(\because nd \cong L$ = Total length of array)

For an end fire array

$$\boxed{D = 4n\left(\frac{d}{\lambda}\right)}$$

$$\Rightarrow D = 4\left(\frac{nd}{\lambda}\right)$$

$$\therefore \boxed{D = 4\left(\frac{L}{\lambda}\right)}$$

$(\because L = (n-1)d$

$\Rightarrow L \cong nd)$

For an end fire array with incre...

$$\boxed{D = 1.789\left[4n\left(\frac{d}{\lambda}\right)\right]}$$

$$\Rightarrow D = 1.789\left[4\left(\frac{nd}{\lambda}\right)\right] \quad (\because L = (n-1)d \quad \therefore L \simeq nd) \text{ if } n \text{ is very large.}$$

$$\therefore \boxed{D = 1.789\left[4\left(\frac{L}{\lambda}\right)\right]}$$

Concept of scanning arrays (or) phased arrays:

→ An array which gives maximum radiation in any direction by controlling phase excitation in each element. Such an array is commonly called "phased array".

→ The array in which the phase and the amplitude of most of the elements is variable.

→ We get the direction of maximum radiation and pattern shape along with side lobes is controlled is called "phased array".

Let the array gives maximum radiation in $\theta = \theta_0$ direction.

$$\therefore \psi = \beta d \cos\theta + \alpha$$

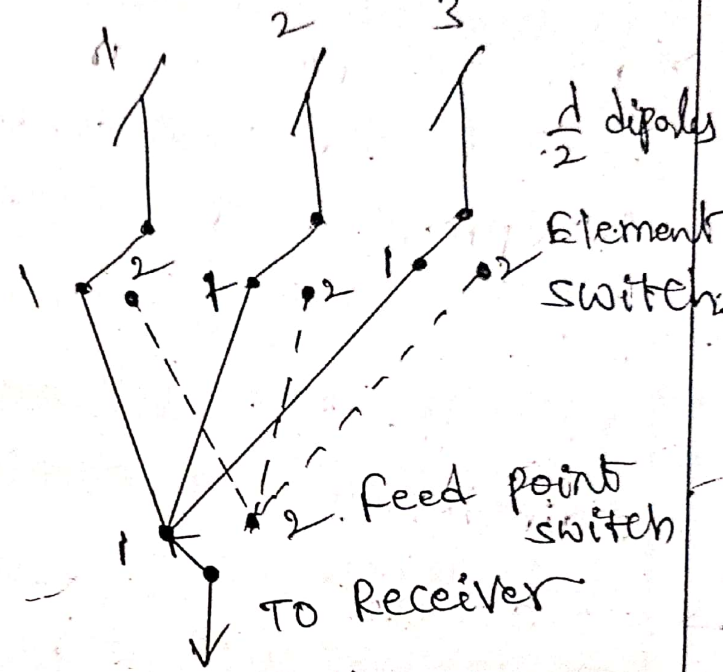
at $\psi = 0$, the radiation is maximum

$$0 = \beta d \cos\theta_0 + \alpha \qquad \text{where } 0 \le \theta_0 \le \pi$$

$$\therefore \boxed{\alpha = -\beta d \cos\theta_0}$$

From above equation, the maximum radiation can be achieved in any direction if the progressive phase difference between the elements is controlled.

→ Let us consider a three element array. the elements of array is considered as $\frac{\lambda}{2}$ dipole.



→ All the cables are of same length.

→ All the cables are taken together at common feed point

→ The mechanical switches are used, one switch at each antenna, and one switch at a common feed point.

→ By operating switch, the beam can be shifted to any phase shift.

Binomial arrays :-

→ In the binomial arrays, the amplitudes of radiating sources are arranged according to coefficients of binomial series.

$$(a+b)^{n-1} = a^{n-1} + \frac{(n-1)}{1!} a^{n-2} b^1 + \frac{(n-1)(n-2)}{2!} a^{n-3} b^2$$
$$+ \frac{(n-1)(n-2)(n-3)}{3!} a^{n-4} b^3 + \cdots$$

Where n = no. of radiating sources in this array.

→ Binomial array can be defined as it is an array in which the amplitudes of the antenna elements are arranged according to the coefficients of the binomial series.

→ For uniform linear array, the array length is increased to increased the directivity. ($\because$ spacing is $\frac{\lambda}{4}$)

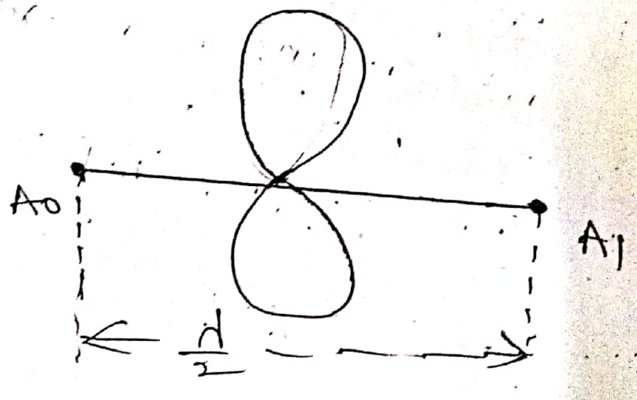
→ But for some applications the secondary lobes should be eliminated, with respect to main lobes.

→ To achieve such a pattern the array is arranged in such a way that broad side array radiate more strongly at the centre.

Let us consider array of two identical point sources spaced $\frac{\lambda}{2}$ apart.

The far field pattern is given by

$$\boxed{E = \cos\left(\frac{\pi}{2}\cos\theta\right)}$$



Dis advantages of binomial array :-

HPBW increases and hence the directivity decreases.

* For design of a large array, larger amplitude ratio of sources is required.

Effect of uniform and Non-uniform amplitude distributions :-

In the design of linear inphase antenna arrays of non-uniform amplitudes C.L Dolph used the Tchebyscheff polynomial, the name is "Dolph-Tchebyscheff arrays"

It is also called as "Chebyshev arrays" (or) "Dolph-chebyshev arrays"

C.L Dolph proposed that for a linear broad-side arrays, it is possible to minimize the beam width of main lobe for a specified side lobe level, Vice Versa

That means If the beam width between first nulls is specified then the side lobe level is minimized.

The current distribution that produce such a pattern is called "Dolph-Tchebyscheff distribution"!

* Therefore Dolph-Tchebyshev distribution provides compromise optimum value between two conflicting properties.

* According to C.L Dolph, the current distribution is optimum provided that distance between two array elements $d \leq \frac{\lambda}{2}$

* For practical design of array, the narrow beamwidth for side lobe levels upto 20-30 dB in UHF and VHF bands.

* 20 dB level is considered for good, 30 dB level considered for excellent. But very difficult to 40 dB level and not exist.

Tchebyscheff polynomials :- (Chebyshev arrays)

The Tchebyscheff polynomial with variable x is denoted by $T_m(x)$.

It is given by

$$T_m(x) = \cos(m\cos^{-1}x), \quad -1 < x < 1 \rightarrow 1(a)$$

$$T_m(x) = \cosh(m\cosh^{-1}x), \quad |x| > 1 \rightarrow 1(b)$$

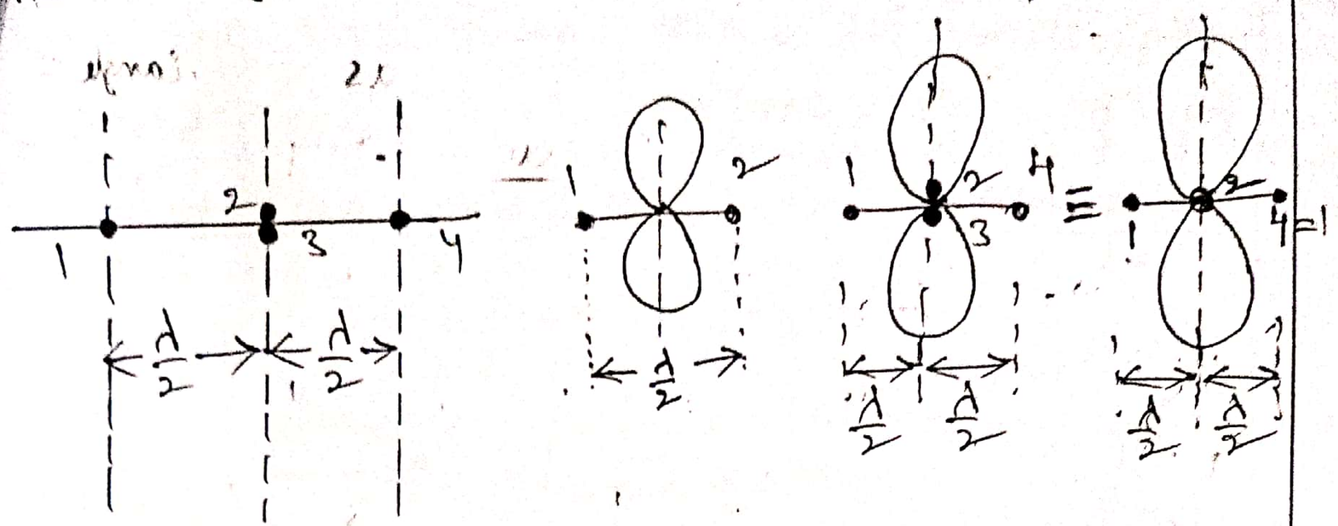
where 'm' is an integer constant from '0' to ∞.

Let m=0. Then eqn 1(a) becomes

$$T_0(x) = \cos(0 \cdot \cos^{-1}x)$$

$$\boxed{\begin{array}{l}\text{Let } \delta = \cos^{-1}x \\ \Rightarrow x = \cos\delta\end{array}}$$

The arrangement of 4-elements with $\frac{\lambda}{2}$ Spacing :-



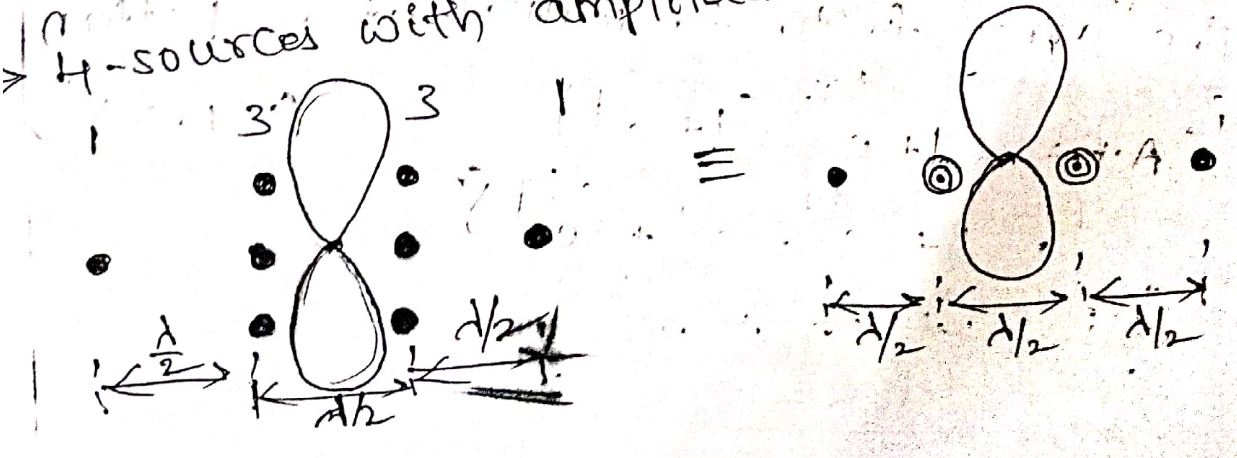
fig(a) arrangement of 4-elements with $\frac{\lambda}{2}$ spacing

fig(b) :- pattern for 2-element array and 4-element array

→. In the uniform 4-element array, the resultant pattern shows four side lobes. the secondary lobes appear in resultant pattern, because the pattern have a spacing ~~between~~ greater than $\frac{\lambda}{2}$.

• so if we reduce the spacing between the two elements to $\frac{\lambda}{2}$ wavelength then the side lobes are reduced, and only main lobes are obtained.

→ 4-sources with amplitude ratio is 1:3:3:1



The binomial series coefficients are ob. by pascal's triangle.

pascal's triangle

no. of sources	Relative amplitude
n=1	1
n=2	1 1
n=3	1 2 1
n=4	1 3 3 1
n=5	1 4 6 4 1
n=6	1 5 10 10 5 1
n=7	1 6 15 20 15 6 1
n=8	1 7 21 35 35 21 7 1
n=9	1 8 28 56 70 56 28 8 1

∴ The pattern for binomial array is given by

$$\boxed{E = \cos^{n-1}\left[\frac{\pi}{2}\cos\theta\right]}$$

The array factor is given by

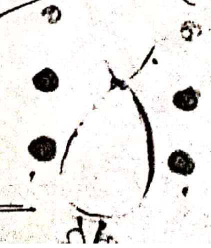
$$A.F = (1+e^{j\psi})^{N-1}$$

The array factor for multiplication pattern is

$$A.F = (1+e^{j\psi})(1+e^{j\psi}) \quad (\because N=3)$$

$$= 1+e^{j\psi}+e^{j\psi}+(e^{j\psi})^2$$

$$\boxed{AF = 1+2e^{j\psi}+e^{j2\psi}}$$



$T_0(x) = \cos(0 \cdot \delta) = \cos 0$

$\therefore \boxed{T_0(x) = 1}$

Let $m=1$:-

$T_1(x) = \cos(1 \cdot \cos^{-1}x) = \cos\delta$

$\therefore \boxed{T_1(x) = x}$ $\left(\because \delta = \cos^{-1}x,\ x = \cos\delta\right)$

Let $m=2$

$T_2(x) = \cos(2 \cdot \cos^{-1}x) = \cos 2\delta$

$= 2\cos^2\delta - 1.$ $(\because \cos 2\theta = 2\cos^2\theta - 1)$

$\boxed{T_2(x) = 2x^2 - 1.}$

Let $m=3$:-

$T_3(x) = \cos(3 \cdot \cos^{-1}x) = \cos 3\delta$

$= 4\cos^3\delta - 3\cos\delta$ $(\because \cos 3\theta = 4\cos^3\theta - 3\cos\theta)$

$\boxed{T_3(x) = 4x^3 - 3x}$

Let $m=4$:-

$T_4(x) = \cos(4 \cdot \cos^{-1}x) = \cos 4\delta$

$= \cos 2(2\delta)$

$= 2\cos^2(2\delta) - 1$

$= 2\left[2\cos^2\delta - 1\right]^2 - 1$ $(\because x = \cos\delta)$

$= 2\left[4\cos^4\delta + 1 - 4\cos^2\delta\right] - 1$

$= 8x^4 + 2 - 8x^2 - 1$

$\therefore \boxed{T_4(x) = 8x^4 - 8x^2 + 1}$

For higher values of 'm' can be calculated by using recursive formula

$$\boxed{T_{m+1}(x) = 2x\,T_m(x) - T_{m-1}(x)}$$

To obtain $T_5(x)$ put $m=4$ in above expression

$\therefore T_{4+1}(x) = T_5(x) = 2x\,T_4(x) - T_3(x)$

$\Rightarrow T_5(x) = 2x[8x^4 - 8x^2 + 1] - [4x^3 - 3x]$

$= 16x^5 - 16x^3 + 2x - 4x^3 + 3x$

$\therefore \boxed{T_5(x) = 16x^5 - 20x^3 + 5x}$

To obtain $T_6(x)$ put $m=5$ in Recursive formula

$T_6(x) = 2x\,T_5(x) - T_4(x)$

$= 2x[16x^5 - 20x^3 + 5x] - [8x^4 - 8x^2 + 1]$

$= 32x^6 - 40x^4 + 10x^2 - 8x^4 + 8x^2 - 1$

$\therefore \boxed{T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1}$

To obtain $T_7(x)$ put $m=6$ in Recursive formula

$T_7(x) = 2x\,T_6(x) - T_5(x)$

$= 2x[32x^6 - 48x^4 + 18x^2 - 1] - [16x^5 - 20x^3 + 5x]$

$= 64x^7 - 96x^5 + 36x^3 - 2x - 16x^5 + 20x^3 - 5x$

$\therefore \boxed{T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x}$

∴ The polynomials are given below

$T_0(X) = 1$ $\quad m = 0$

$T_1(X) = X$ $\quad m = 1$

$T_2(X) = 2X^2 - 1$ $\quad m = 2$

$T_3(X) = 4X^3 - 3X$ $\quad m = 3$

$T_4(X) = 8X^4 - 8X^2 + 1$ $\quad m = 4$

$T_5(X) = 16X^5 - 20X^3 + 5X$ $\quad m = 5$

$T_6(X) = 32X^6 - 48X^4 + 18X^2 - 1$ $\quad m = 6$

$T_7(X) = 64X^7 - 112X^5 + 56X^3 - 7X$ $\quad m = 7$

$T_8(X) = 128X^8 - 256X^6 + 160X^4 - 32X^2 + 1$; $m = 8$

$T_9(X) = 256X^9 - 576X^7 + 432X^5 - 120X^3 + 9X$; $m = 9$.

Therefore the degree of Tchebyshev polynomial is same as value of 'm'. It is either even or odd.

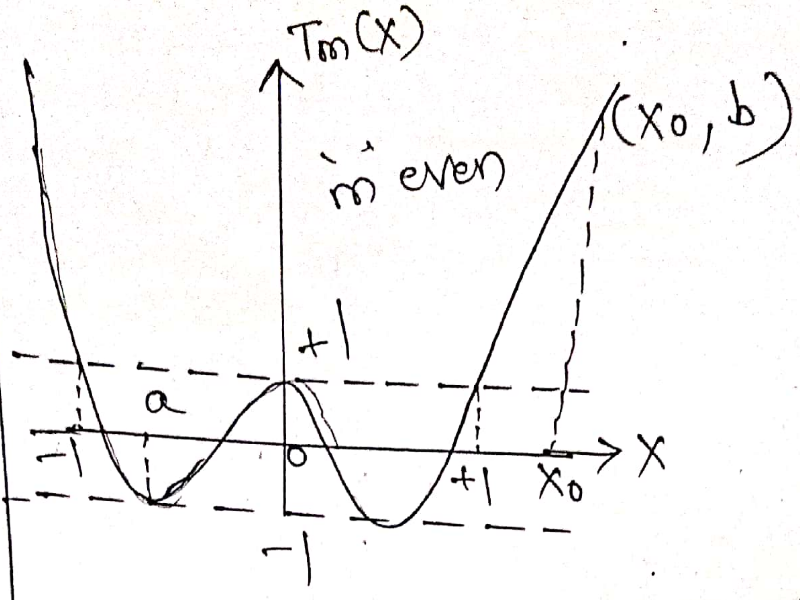
<u>properties</u> :-

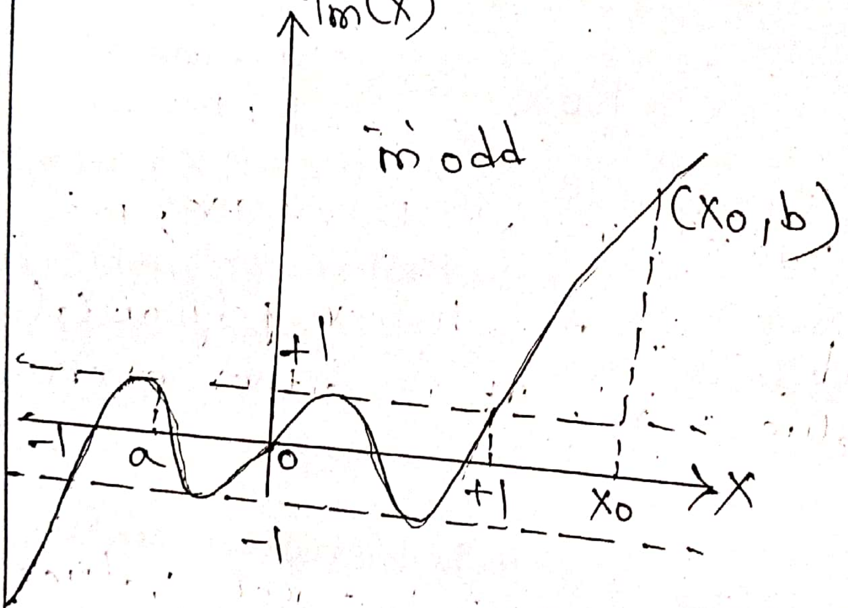
∴ All the polynomials oscillate between values -1 and 1

In the region $|x| < 1$; the m^{th} order polynomial crosses the axis 'm' times.

In the region $|x| > 1$, the Tchebyshev polynomial go on increasing.

The Tchebyshev polynomial waveforms are given by





The mth order polynomial is $T_m(x) = \cos(m \cos^{-1} x)$

The nulls are given by the roots.

$$\cos(m \cos^{-1} x) = 0$$

$$\Rightarrow \cos(m\delta) = 0$$

$$m\delta = \cos^{-1}(0)$$

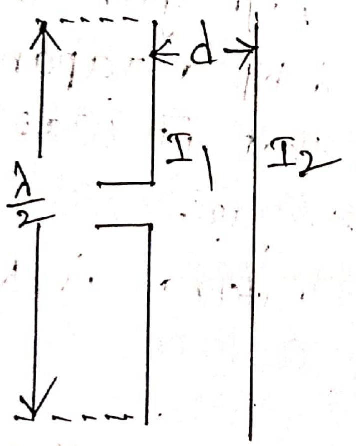
$$m\delta = (2k-1)\frac{\pi}{2}, \quad k = 1, 2, 3 \ldots m$$

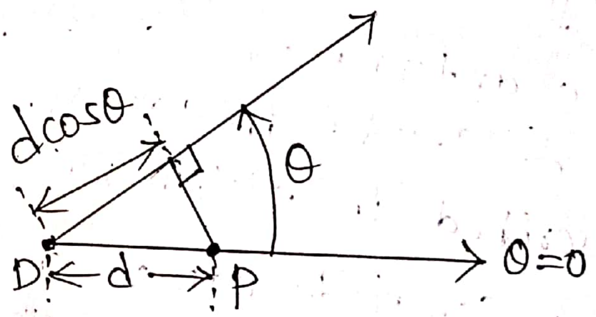
$$\boxed{\delta = \frac{(2k-1)\pi}{2m}} \quad k = 1, 2, 3 \ldots m$$

Arrays with parasitic elements:-

The element in which current is induced due to the field in other elements is called as "parasitic element".

* One (or) more parasitic elements coupled magnetically with the driven element forms an "array of parasitic elements".
* It is also called as parasitic antenna.
* The effect of parasitic element on the directional pattern of the antenna depends on the magnitude and the phase of the induced current in parasitic element.



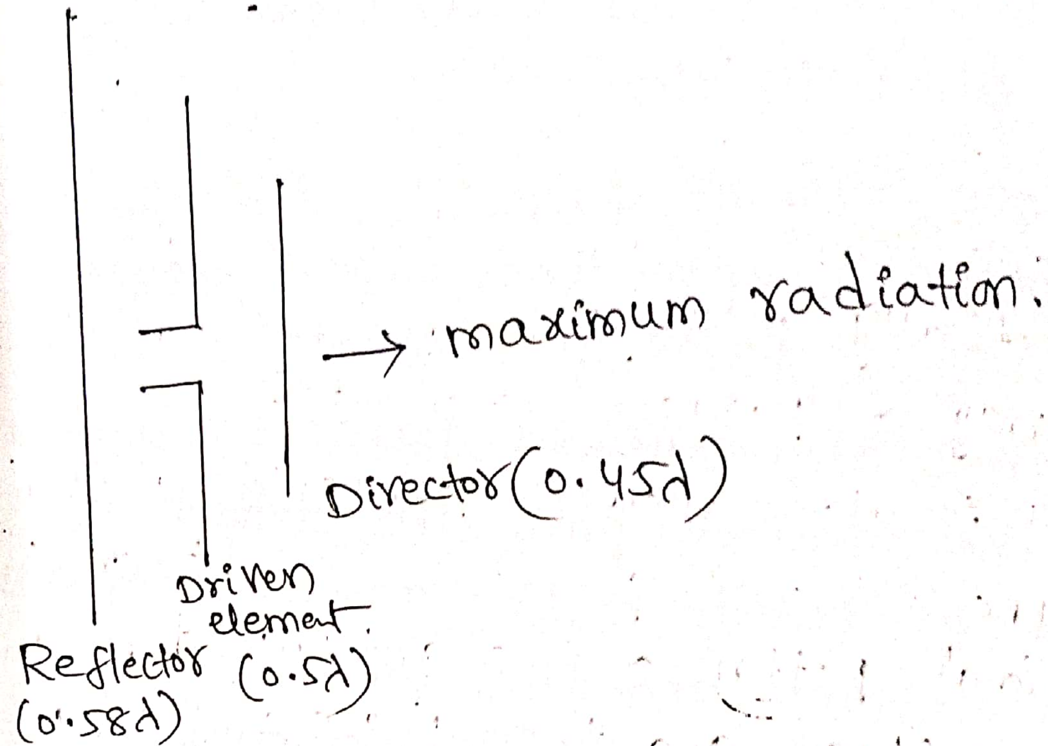


D = Driven element

P = parasitic element.

* When the parasitic element is larger than its resonant ($\frac{\lambda}{2}$) length, it is inductive in nature. Then such element is acts as "reflector".
* When the parasitic element is shorter than its resonant ($\frac{\lambda}{2}$) length. it is capacitive in nature then such element is acts as "Director".

The 3-element parasitic array is given below

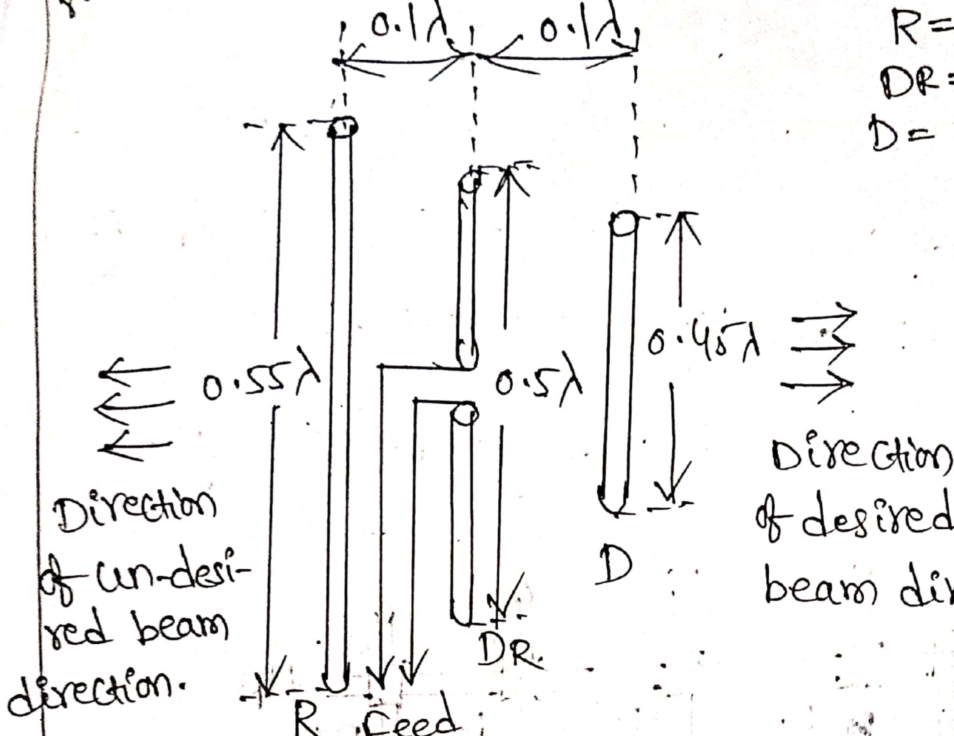


Yagi-Uda Array :- (or) Yagi-uda antenna

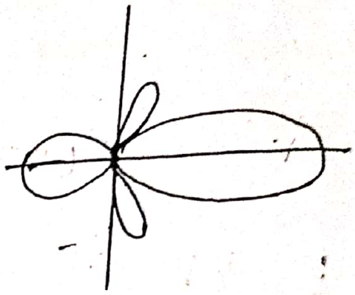
* Yagi-uda antennas are most high gain antenna
* The antenna was first invented by a Japanese prof. S. Uda in 1940's.. after that, it was described in english by prof. H-Yagi.
* The complete name of this antenna is known as "Yagi-Uda antenna".
* It consists of a driven element, a reflector, and one (or) more directors.
* That is Yagi-uda antenna is an array of a driven element and one (or) more parasitic elements

Dess the driven element is a resonant half wave dipole usually of metallic rod at frequency of operation.

* the parasitic elements are continuous metallic rods arranged in parallel to driven element.



fig(a) :- Yagi-uda array



fig(b) Radiation pattern.

* The parasitic element Receive their excitation from voltages induced in these elements by the current flow in the driven element.

* Generally the spacing between driven element and parasitic elements is 0.1 λ to 0.15 λ.

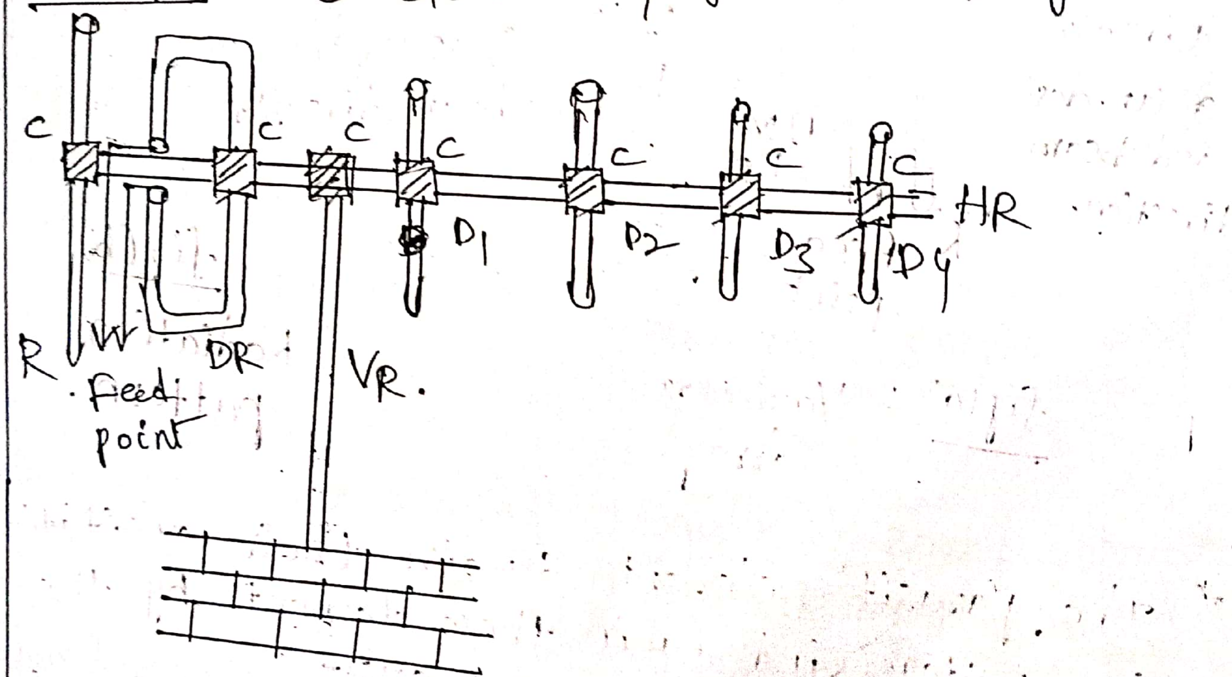
Reflector length $= \frac{152}{f(MHz)}$ meter (∴ 1 meter = 3.3 ft)

Driven element length $= \frac{143}{f(MHz)}$ meter

Director length $= \frac{137}{f(MHz)}$ meter

* The spacing between elements and lengths of the parasitic elements determines the phases of the currents.
* A parasitic element of length greater than $\frac{\lambda}{2}$ then it is Inductive and is called as "Reflector"
* A parasitic element of length less than $\frac{\lambda}{2}$ then it is capacitive and is called as "Director".
* The element of length is equal to $\frac{\lambda}{2}$ then it is driven element (or) dipole element.

Example :- 6-element Yagi-uda array.



Where R = Reflector
DR = driven element = folded dipole
D_1, D_2, D_3, D_4 = directors
VR = Vertical rod to support horizontal rod
HR = horizontal rod to support elements
C = clamps.

NON-RESONANT RADIATORS:-

* The antennas which operate between frequency range of 3-30 MHz are called high-frequency antennas.
* For the HF band, the wavelength ranges in 100m to 10m.
* So the HF antennas can be made in size comparable with the wavelength.
* The directional properties can also be obtained for such antennas.
* In case of Low frequency and ~~high~~ medium frequency band, the wavelength is greater, the size of antenna becomes larger, and it becomes difficult to achieve highly directive system.

Resonant Antenna:-

→ Resonant antennas are antennas which correspond to transmission line and standing waves are exist. (Incident waves + Reflected waves)

* the resonant antennas are also called as periodic antennas
* Examples of Resonant antennas are half wave dipoles, quarter wave monopoles, folded dipoles.
* The radiation pattern is bi-directional.

Non-Resonant antenna :-

* The Non-Resonant antennas are antennas which is also corresponds to transmission line, but there is no standing waves occurs. Because it exists only travelling waves (Incident wave).

* In the HF band vertical radiators is not a suitable choice. so practically the simplest antenna that can be used is horizontal antenna $\frac{\lambda}{2}$ dipole.

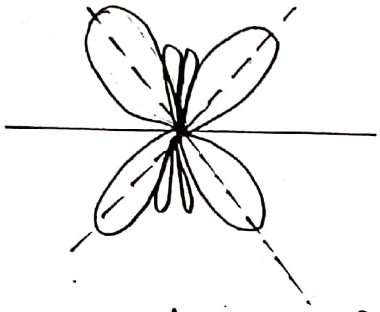
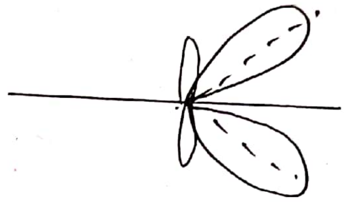
* The Non-Resonant radiator (or) antenna is also called as "aperiodic" antenna.

* Examples are travelling wave antennas, long wire antenna, V-antenna, inverted V-antenna, Rhombic antenna.

* The Radiation pattern is uni-directional.

Comparision between Resonant and Non-Resonant antennas :-

Resonant Antenna	Non-Resonant Antenna
* Resonant antennas are antennas having exact no. of $\frac{\lambda}{2}$ wavelength long, and open at both the ends.	→ It is a transmission line excited at one end and properly terminated at other end.

In this antenna, the standing waves are exist It has bi-directional radiation pattern  → It operates at fixed frequency	→ No standing waves are exist due to No Reflected waves. → It has uni-directional radiation pattern.  → It operated at various types of frequencies.

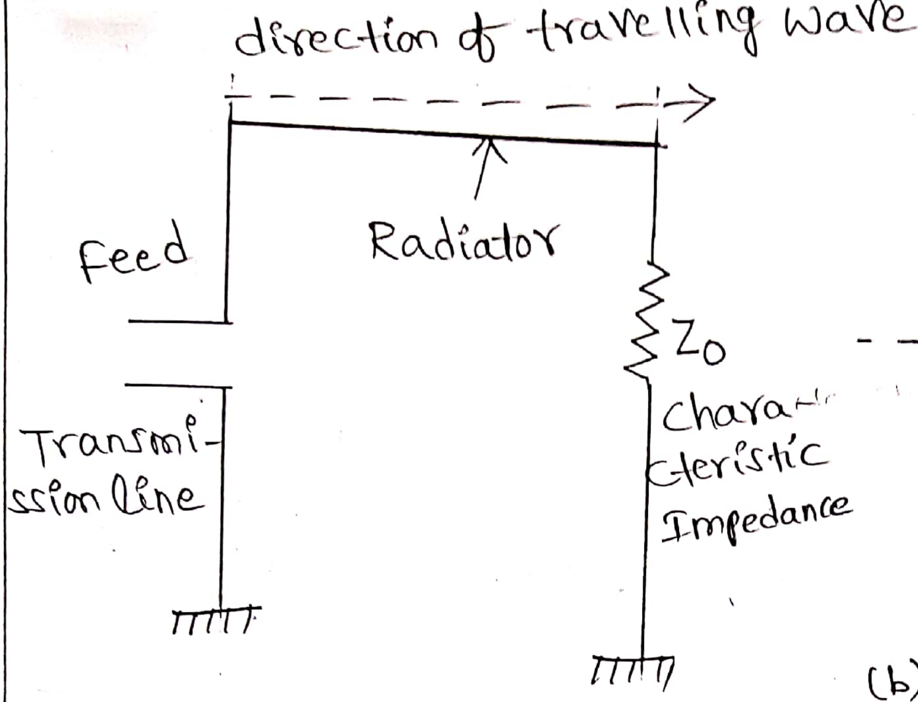
Travelling wave Radiators:- [Travelling wave Antennas].

* The antenna in which the standing waves does not exist along the length of the antenna this is called as "travelling wave antenna".

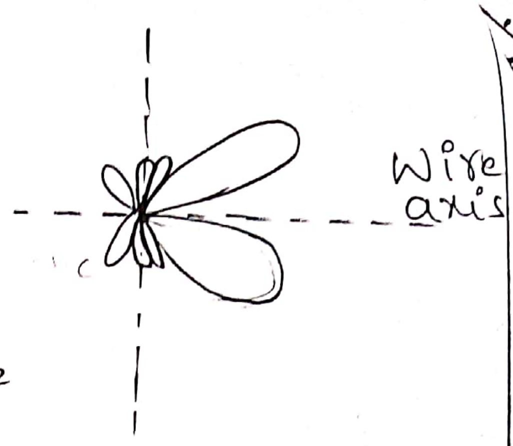
* Generally the standing waves exist when the Antenna wire is not properly terminated which causes reflections are produced at load.

* Therefore the standing waves exist in the Resonant antenna and not exist in the non-Resonant antenna

* In travelling wave antenna no reflections are produced. due to which No standing waves occurs.



(a) Travelling wave antenna



(b) uni-directional Radiation pattern.

* The current will change phase with distance that is progressive phase change in the end-fire array case.
* The velocity of light in wire is same as in the freespace.

The strength of the electric field at a distance 'r' is given by

$$\boxed{E = \frac{60\, I_{rms}}{r}\left(\frac{\sin\theta}{1-\cos\theta}\right)\sin\left(\frac{\pi L}{\lambda}(1-\cos\theta)\right)}$$

r = distance at a point from radiator

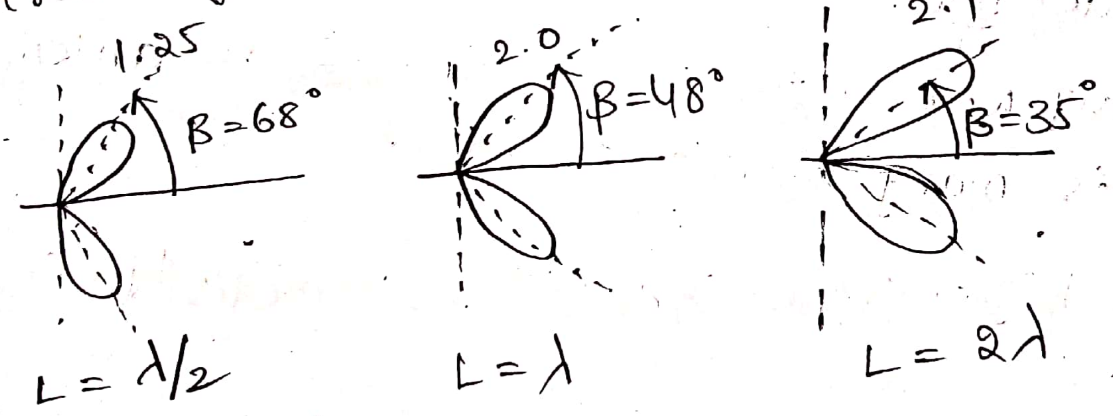
L = Length of wire (or) radiator.

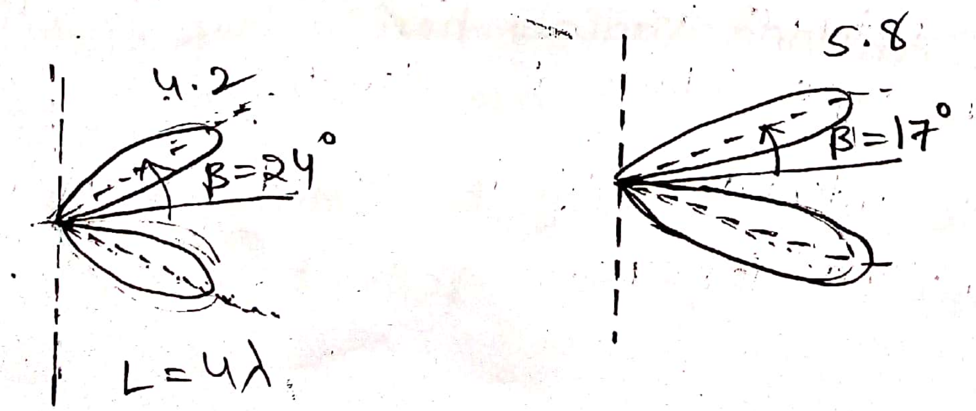
The angle and amplitude of the major lobe depends on the length of the wire.

* If the length of wire increases then the angle of major lobe decreases and amplitude of major lobe Increases.

Length of wire	Angle of major lobe	Amplitude of major lobe
$L = \frac{\lambda}{2}$	68°	1.25
λ	48°	2.0
2λ	35°	2.9
4λ	24°	4.2
8λ	17°	5.8

The radiation pattern for different lengths of travelling wave antenna





Advantages:-

* standing waves doesnot exist
* compared to single wire antenna Band width is more.
* Less power dissipation.
* used in Radio communications, applications.

Dis advantages:-

* The waves can be propagated in only one direction
* Large space requirement.

Long wire antenna:- (Hormonic Antenna)

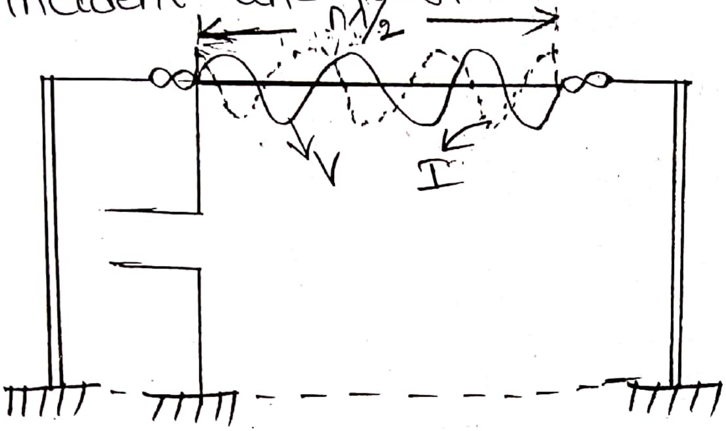
* A long wire antenna is defined as a single long wire, typically n times of $\frac{\lambda}{2}$ long at the operating frequency
* It is also an integer multiple of Half wave length (ie) $\frac{n\lambda}{2}$.
* ∴ A long wire antenna is linear wire antenna which is many wavelength long.
* If the higher value of 'n', the directivity is better.
* A long wire antenna radiates horizontally polarized wave at an angle which are $17°$ to $25°$.
* A long wire antenna may be considered as a resonant antenna (or) non-resonant antenna.

Resonant Long wire Antenna:-

(4)

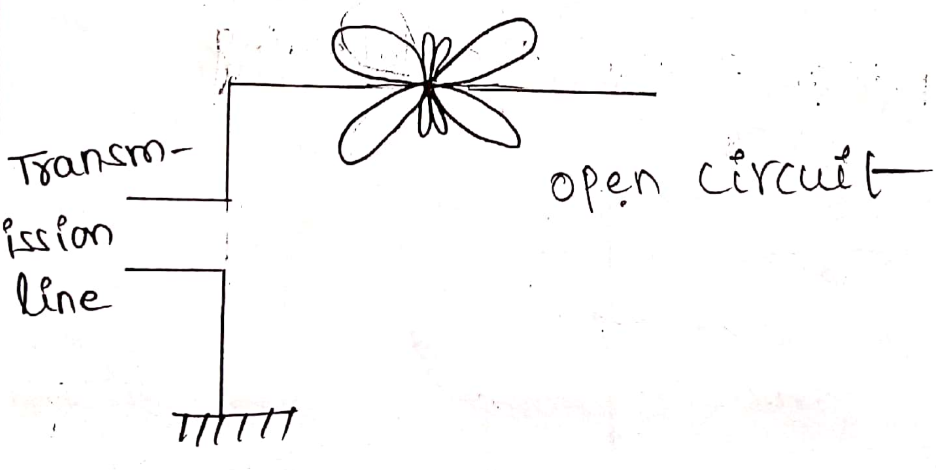
→ In the Resonant Long wire antenna, the load end is open (or) un terminated. There fore the standing waves are observed along the length of antenna.

→ Thus the pattern is bidirectional due to the incident and reflected waves.



Resonant Long wire Antenna

Bidirectional radiation pattern is given the following figure



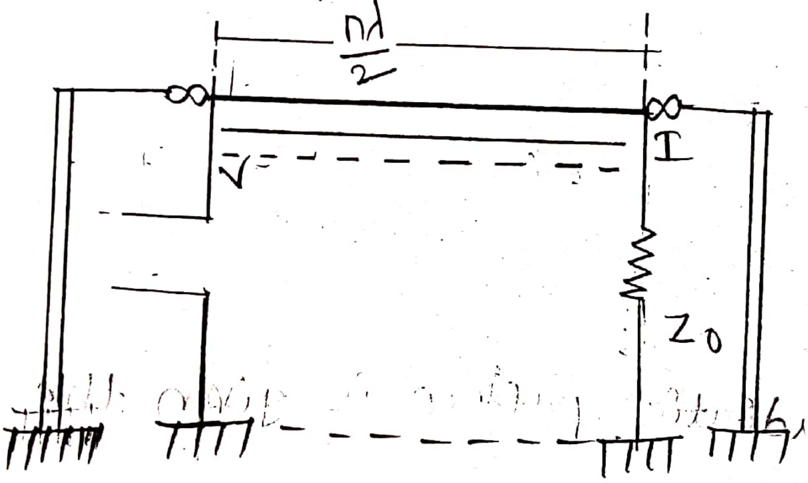
Non - Resonant Long wire Antenna:-

→ In non- Resonant long wire antenna, the load end is terminated with characteristic impedance (or) non inductive resistance.

→ ∴ No standing waves are exist along the length of antenna.

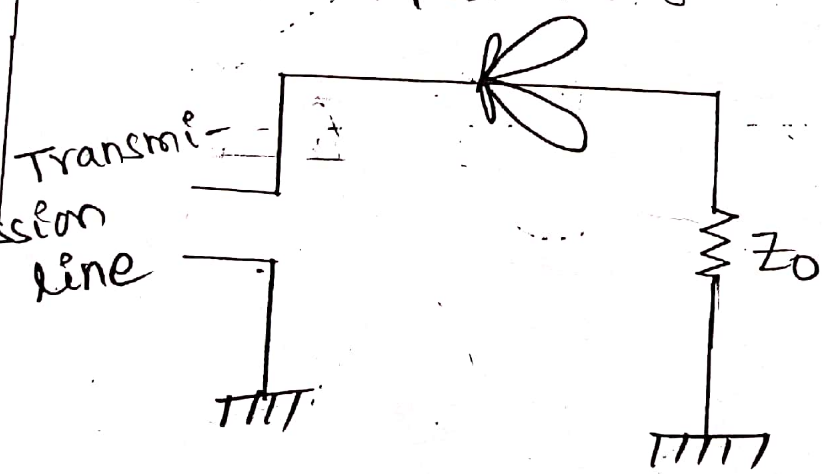
→ All the incident waves are absorbed and no reflections are produced.

→ Thus the pattern is uni-directional.



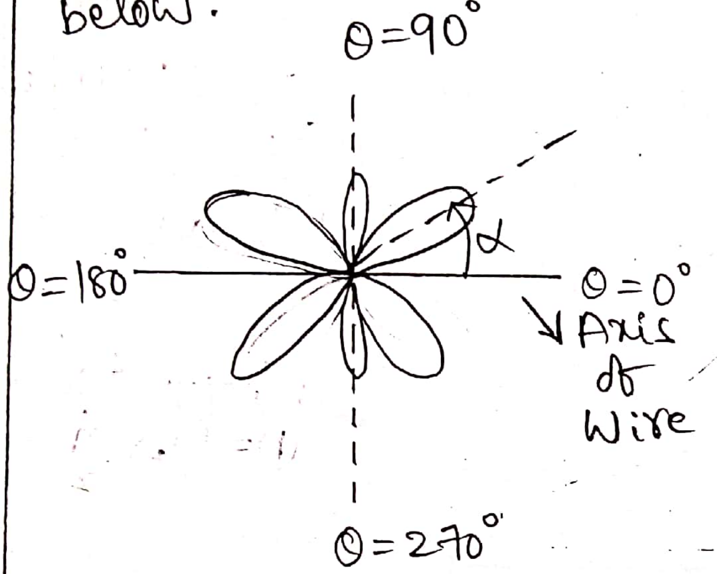
Non Resonant Long wire antenna

Unidirectional pattern is given the following

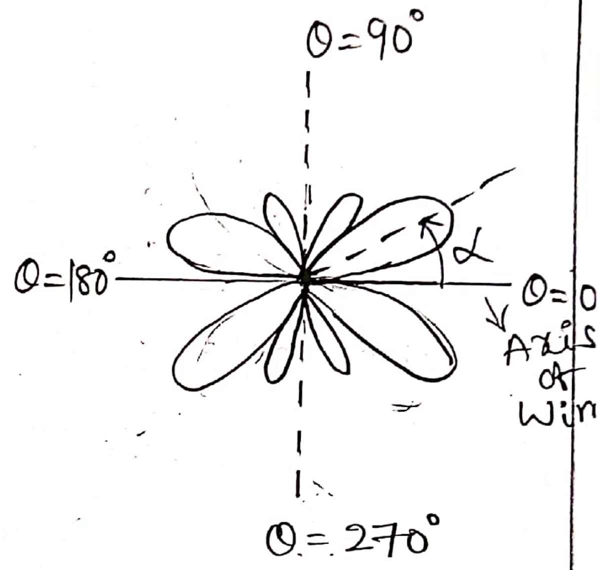


Depending on if 'n' is even and odd, the directional pattern changes.

→ The patterns of long wire antenna for different integer multiples (ie) n=3 and n=4 are given below.



n=3

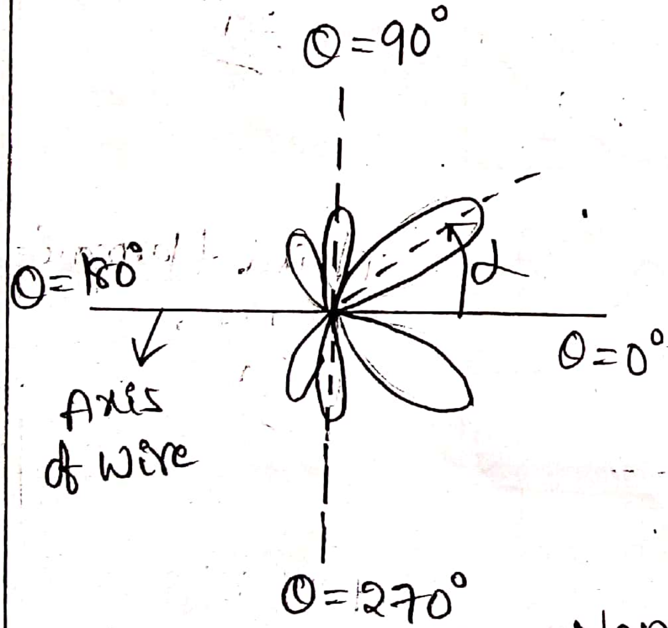


n=4

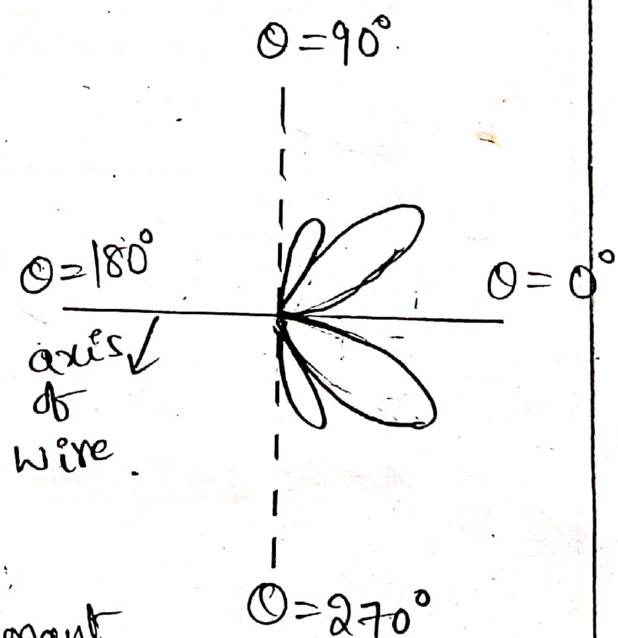
n=3 → odd
n=4 → even

Resonant long wire Antenna.

n-odd $E = \frac{60 I_{rms}}{r} \frac{\cos\left(\frac{n\pi}{2}\cos\theta\right)}{\sin\theta}$



n=3



n=4

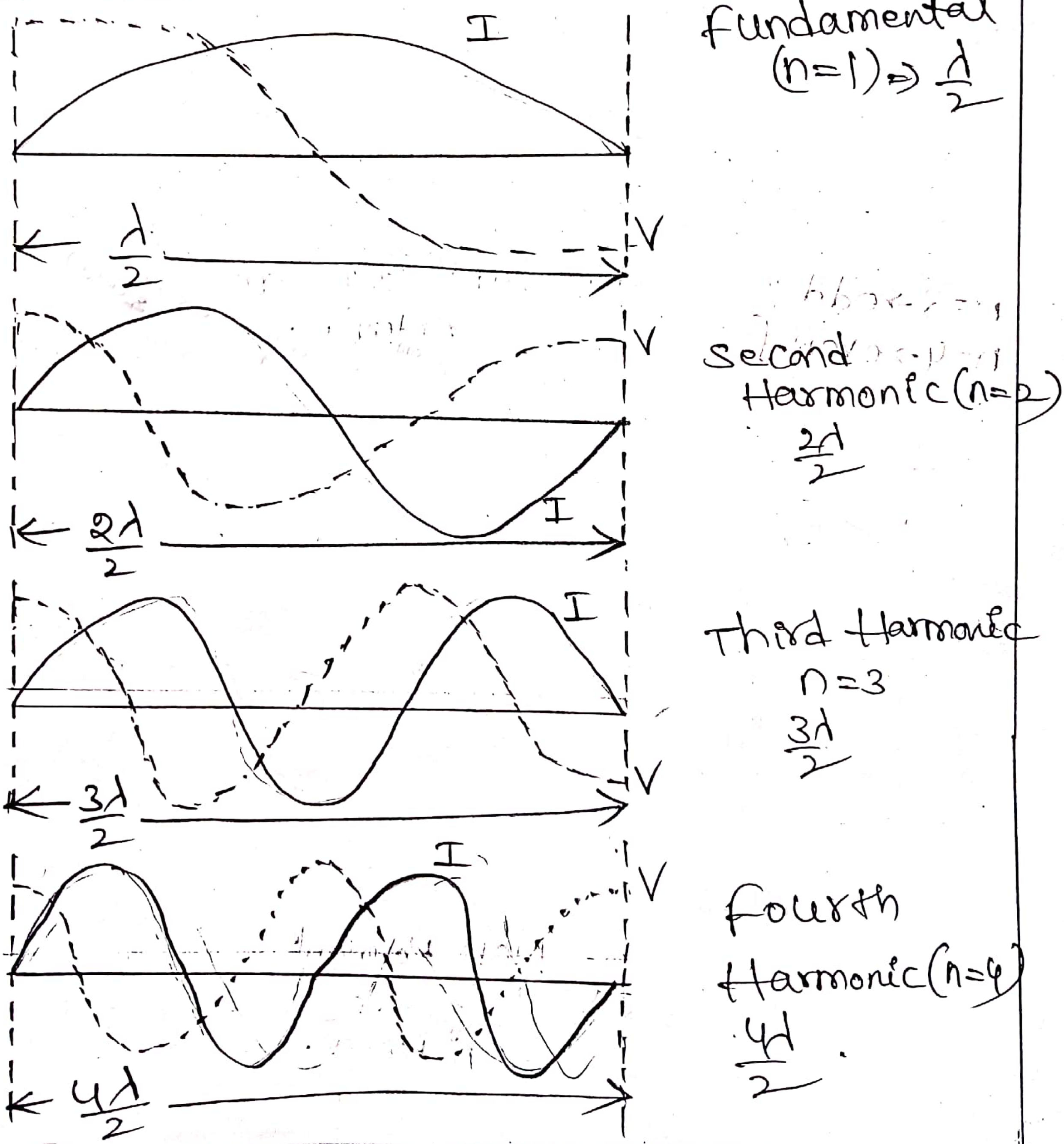
Non Resonant long wire Antenna.

* For the half wavelength long wire antenna, the physical length is given by

$$\boxed{\text{Length} = \frac{492(n-0.5)}{f(MHz)} \text{ feet}}$$

n = no. of integer multiple half wavelength.

→ The Voltage and current distribution along resonant wire working a fundamental frequency ($\lambda/2$) and second harmonic ($\frac{2\lambda}{2}$), 3rd harmonic ($\frac{3\lambda}{2}$) are shown below.



The field strength for resonant long wire antenna with length even and odd integer multiples of $\frac{\lambda}{2}$ are given by

$$\boxed{\begin{aligned} E &= \frac{60\, I_{rms}}{r}\left[\frac{\cos\left(\frac{n\pi}{2}\cos\theta\right)}{\sin\theta}\right] \dashrightarrow n \text{ is odd} \\ E &= \frac{60\, I_{rms}}{r}\left[\frac{\sin\left(\frac{n\pi}{2}\cos\theta\right)}{\sin\theta}\right] \dashrightarrow n \text{ is even.} \end{aligned}}$$

Similarly the field strength for non - resonant long wire antenna is given by

$$\boxed{E = \frac{60\, I_{rms}}{r} \cdot \frac{\sin\theta}{1-\cos\theta} \cdot \sin\left(\frac{\pi L}{\lambda}(1-\cos\theta)\right)}$$

→ When the integer value of 'n' is increased, that increases the no. of lobes in proportion with major lobe.

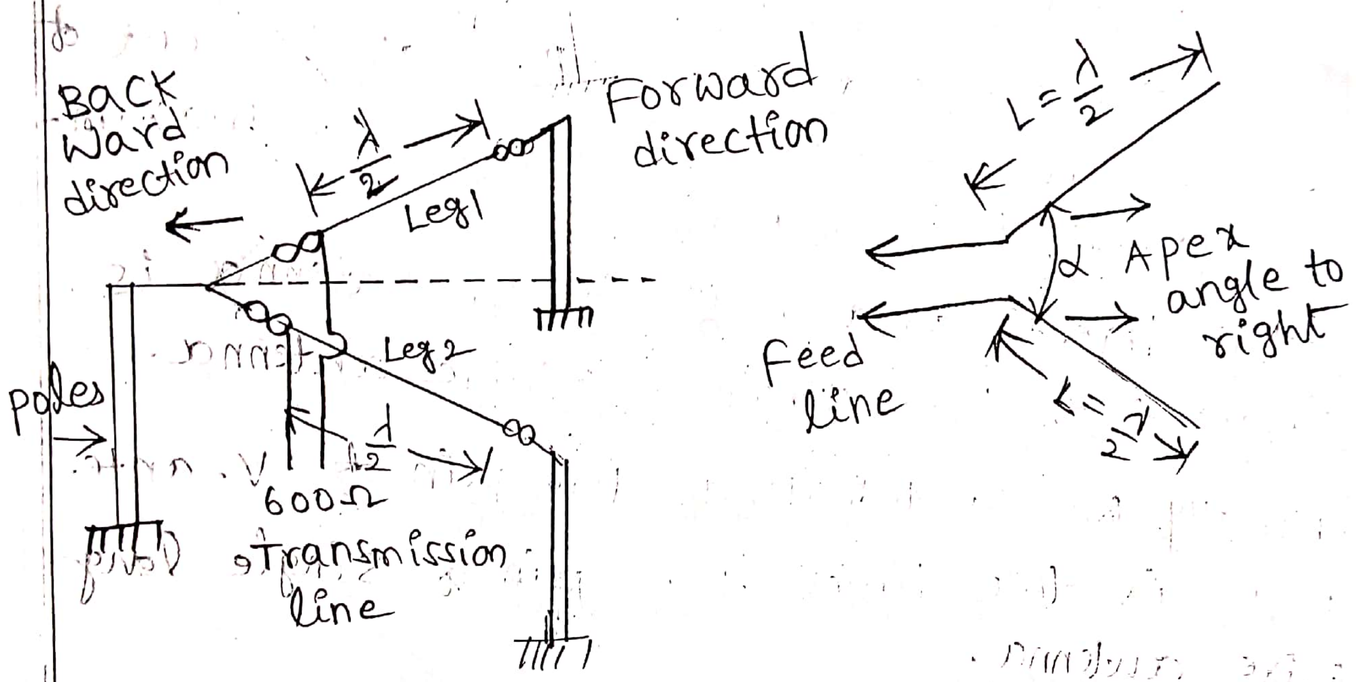
→ For the resonant long wire antenna of 'n' wavelength long, the radiation resistance is $\boxed{R_{rad} = 73 + 69 \log_{10} n \ \Omega}$

→ The angle of maximum radiation is given by $\boxed{\theta_{max} = \cos^{-1}\left(\frac{n-1}{n}\right)}$

V- antenna :-

* The V- antenna is extension of long wire antenna. the two long wires are arranged in the form of horizontal 'V' fed at apex angle.

* The Resonant V- antenna arrangement is given below.

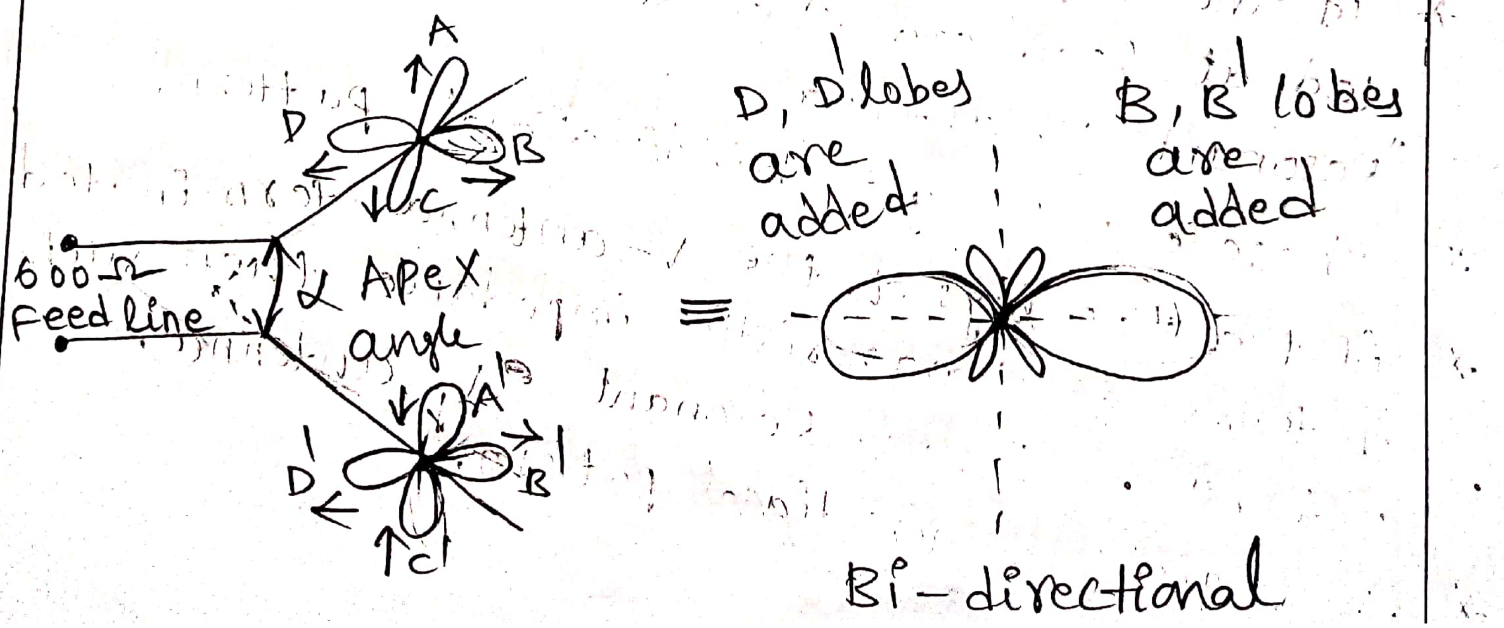


Resonant V- antenna

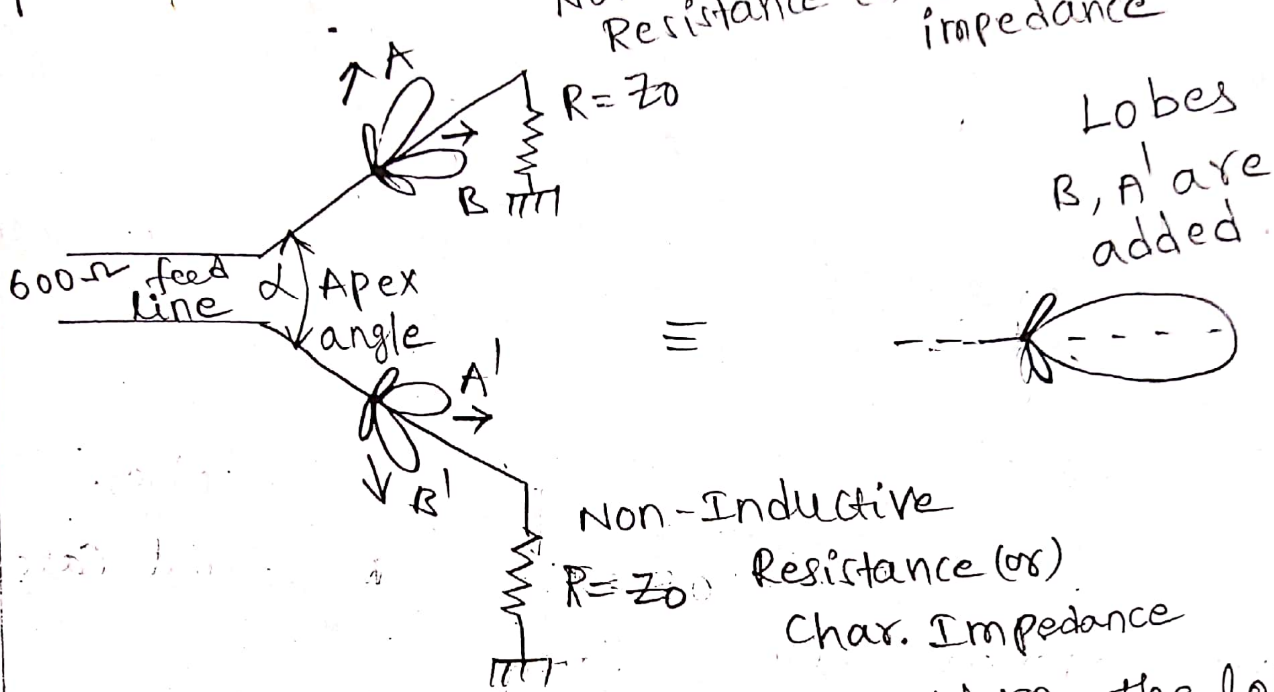
* if the two legs of V- antenna not terminated at load end then such antenna is "Resonant V- antenna.
* Therefore we get bi-directional pattern.
* if the two legs of the V- antenna terminated in terms of characteristic impedance then such antenna is " non - Resonant V- antenna.
* ∴ We get uni - directional pattern.

* The angle between two legs of the V-antenna is called as "apex angle α".
* the apex angle is equal to twice the angle that the cone of maximum direction with ~~with the~~ wires (legs) makes by the axis of V-antenna.
* ∴ The two cone angles are adding to get the maximum radiation.
* The two wires are connected at 180° out of phase with each other. So we get maximum directivity and gain.
* the directivity and gain of V-antenna is larger than the single long wire antenna.
* Finally we conclude that the gain of V-antenna is two times the gain of single long wire antenna.

Resonant V-antenna with bi-directional pattern is



Non-Resonant V-antenna with uni-directional pattern is



* "In the Resonant V-antenna pattern the lobes D and D' are added in the backward direction as they are in the same direction.
* similarly the lobes B and B' are added in the forward direction, as they are also in the same direction.
* The ~~remaining~~ remaining lobes A, A' and C, C' are cancelled due to opposite direction.
* Thus we get bi-directional pattern with increased directivity and gain.
* "In the Non-Resonant V-antenna pattern the lobes B and A' are added in the same direction, A and B' are cancelled.
* Thus we get uni-directional radiation pattern with increased directivity & gain.

Advantages:-

1. It is the cheapest form of Transmitting and Receiving antenna for lower beam.

2. It has high directivity & gain

3. The apex angle varries from 36° to 72° for the length of 8λ to 2λ

Drawbacks:-

1. It produces too many strong side lobes.
2. The band width is less in Resonant case.
3. More expensive [high cost].

Inverted V- antenna:-

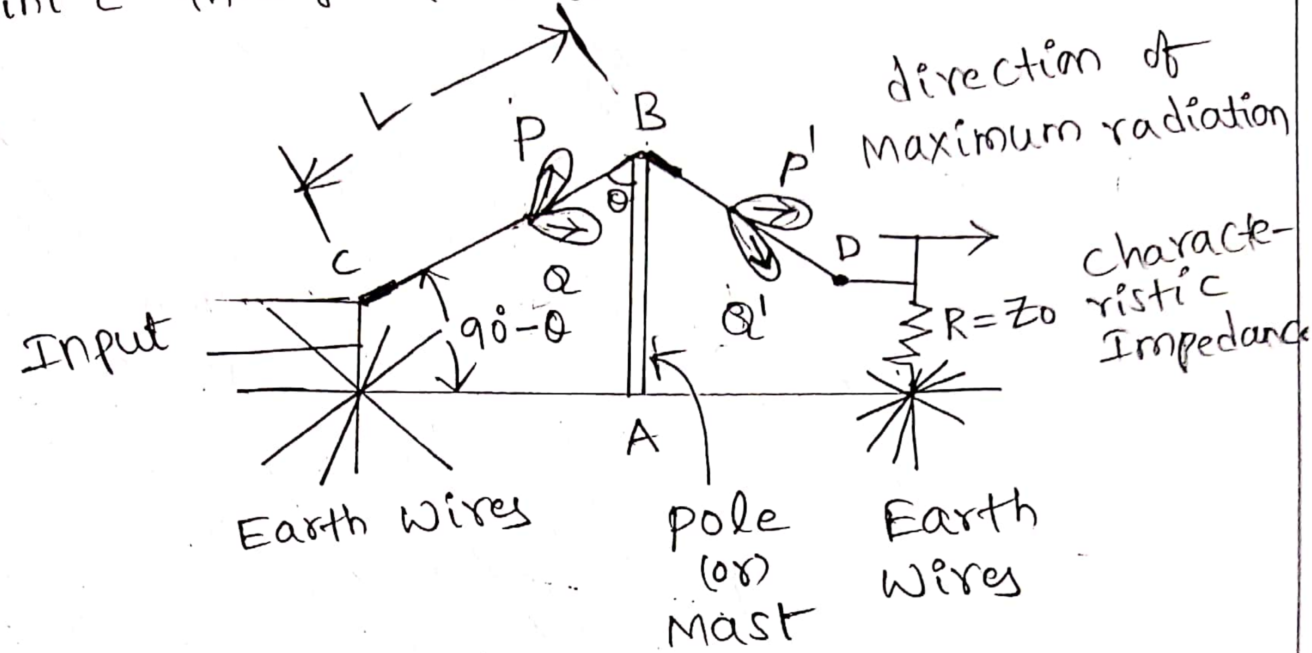
* The resonant (or) Tuned antennas are having small band width and more expensive.
* ∴ Resonant antennas are operated at fixed frequency
* The large band width can be achieved by Travelling wave antennas in which no standing waves produced.
* The inverted V-antenna used in the high frequency band is one of the travelling wave antenna.

The principle and working as given below.

(9)

Principle :-

The input from transmitter is applied at point 'C' through the transmission line.

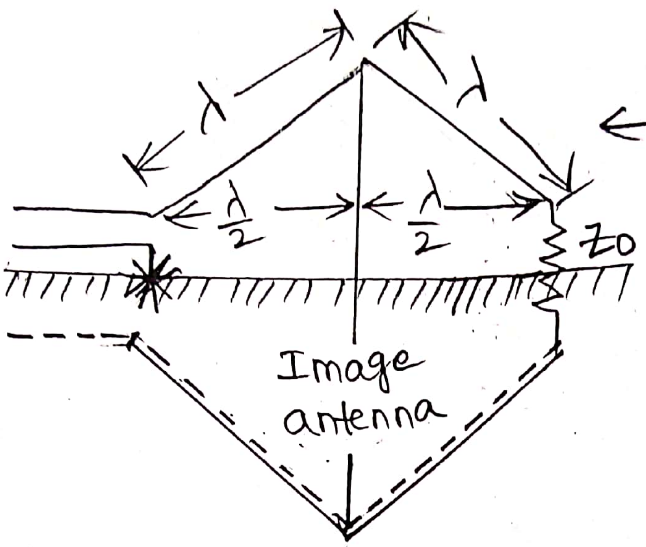


Inverted V-antenna

* One end is connected to no. of radial earth wires, the next end of antenna wire 'D' is connected to earth wires and terminated with characteristic impedance (or) Non-Inductive resistance.
* This resistance is typically 400 Ω and adjusted to set the travelling waves in the BCD (or) CBD wire.
* The angle 'θ' is known as "tilt angle". It is given by
  (i) The angle of major lobe corresponding to $\frac{L}{\lambda}$
  (ii) The angle of tilt, where the fields from BC, BD combined to give Max. gain, directivity

* From above figure the lobes Q, P' are added and P, Q' are cancelled.

Inverted V-earthed antenna is given below.



The inverted V-antenna and its image antenna combinely to give the Rhombic antenna with maximum gain along the ground plane.

* The maximum gain occurs at half of the breaster angle (θB)

* It is the critical angle of incidence in the vertical polarized waves.

Advantages :-

1. The max gain & directivity
2. The Band width is large.
3. It is Better suitable for upper end of high frequency transmission.
4. The signal Receiving is better.

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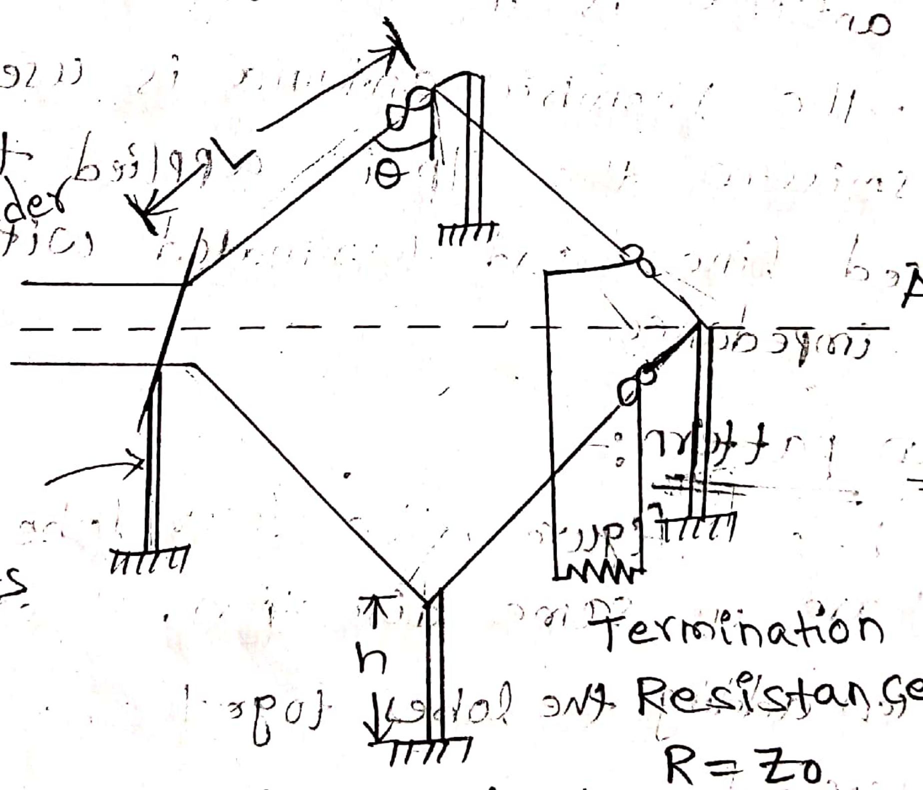
Disadvantages :-

1. It has unwanted side lobes.
2. These side lobes produce horizontal polarized waves.

Rhombic antenna :- [Diamond Antenna]

* A rhombic antenna is equilateral parallelogram, generally with two opposite acute angles.
* The rhombic antenna is based on the principle of travelling wave radiator.
* The two sides are pulled at one point to get the Rhombus (or) diamond shape.



Rhombic Antenna

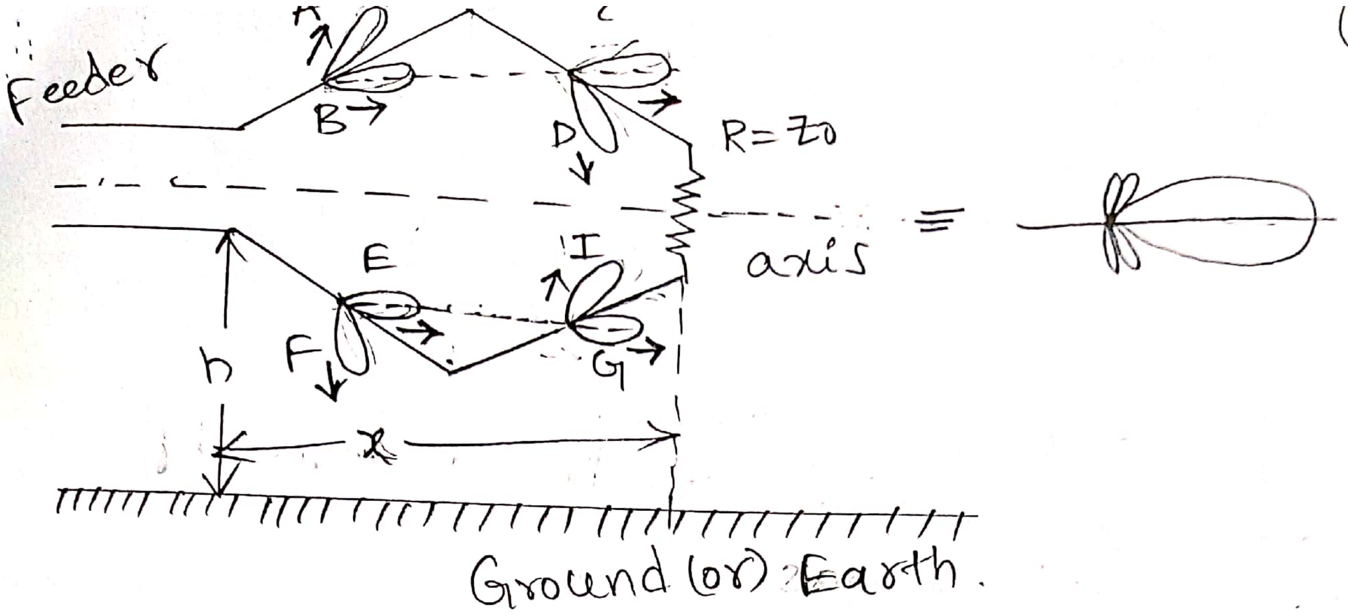
The tilt angle (θt) is approximately equal to (90°−θ). Where 'θ' is angle of major lobe.

* Rhombic antenna consists of four sides are arranged in the form of diamond (or) rhombus.
* The Rhombic antenna is obtained by connecting two inverted V-antennas in parallel.
* The inverted V-antenna and its image antenna gives Rhombic antenna.
* Rhombic antenna is installed on the horizontally over the ground at height of h'
* The polarized waves from the horizontal Rhombic antenna is in the rhombus plane.
* Whenever, the rhombic antenna is used for the transmission, the i/p is applied through a balanced line and terminated with characteristic impedance.

Radiation pattern:-

* In the below figure, the four lobes B, C, E, G all are in same direction. therefore combine (or) adding the lobes together.
* The additional gain is achieved.
* the lobes A, D, F, I are in opposite direction and cancelled these lobes.

∴ We get unidirectional radiation pattern.



Advantages:-

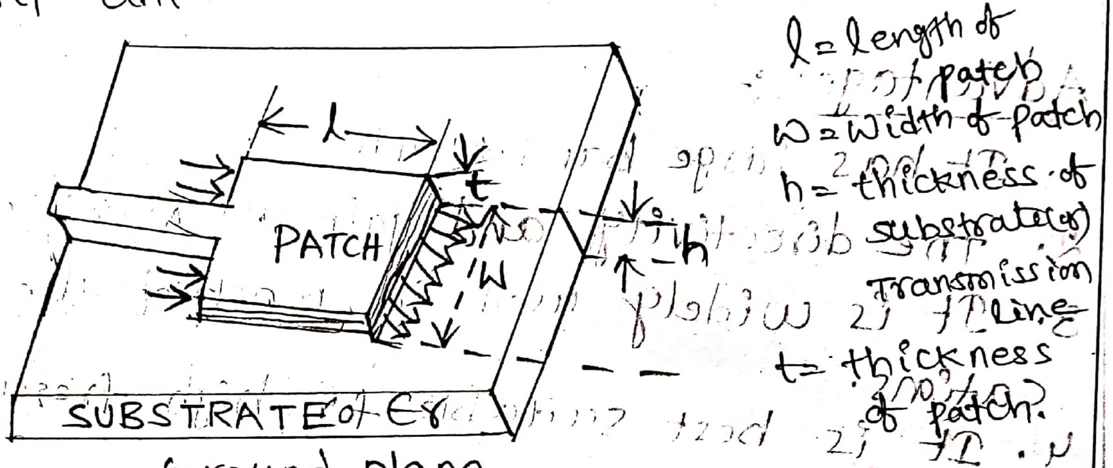
1. It has large band width
2. The directivity and gain is maximum
3. It is widely used in most of the communications.
4. It is best suitable for high frequency Transmission and Reception.

disadvantages:-

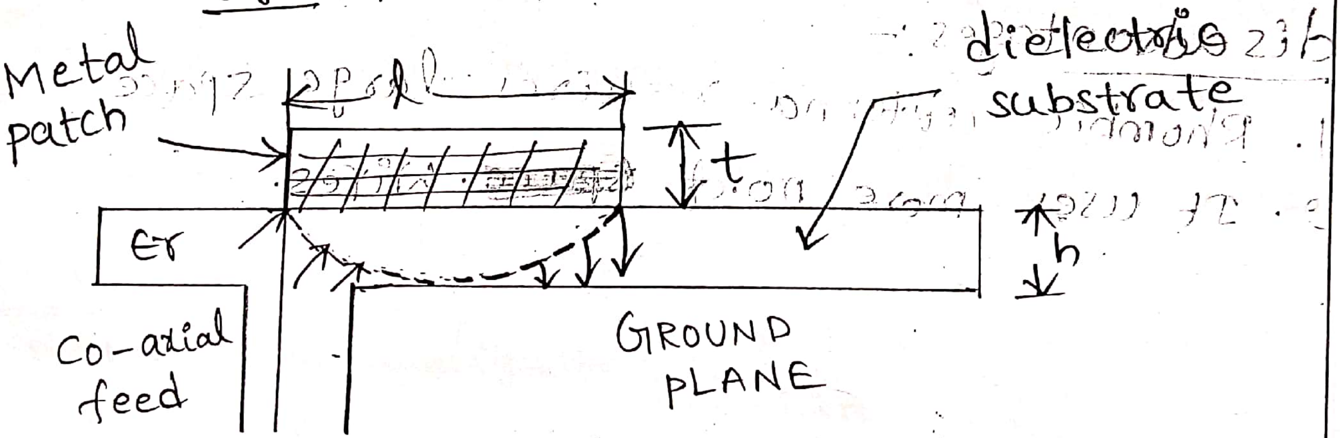
1. Rhombic antenna requires large space
2. It uses more no.of ~~space~~ wires.

Micro strip antennas :- [patch Antennas].

* Micro strip antennas are also called as patch antennas (or) printed antennas.
* The specifications required for aircraft applications are size, weight, cost, performance, easy of installation.
* These specifications can be achieved by "micro strip" antennas.



fig(a) :- patch (or) Micro strip antenna



fig(b) :- side view of patch antenn (or) Micro strip.

* The micro strip antennas are usually used for the low profile applications at a frequencies above 100 MHz with $\lambda < 3m$. wavelength.

Q) The Radiation Conductance is

$$G = \frac{1}{90}\left(\frac{W}{\lambda}\right)^2 \quad \text{if } W << \lambda$$

$$G = \frac{1}{120}\left(\frac{W}{\lambda}\right) \quad \text{if } W >> \lambda .$$

Advantages :-

1. Low fabrication cost
2. High performance
3. Low cost
4. Less weight
5. It supports both linear and circular polarization.
6. It operates on dual and triple frequencies
7. Less size.

Dis advantages :-

1. Narrow Bandwidth
2. gain is low (6 db)
3. Less efficiency

Remedy :- The bandwidth can be increased by increasing thickness 'h' of dielectric substrate

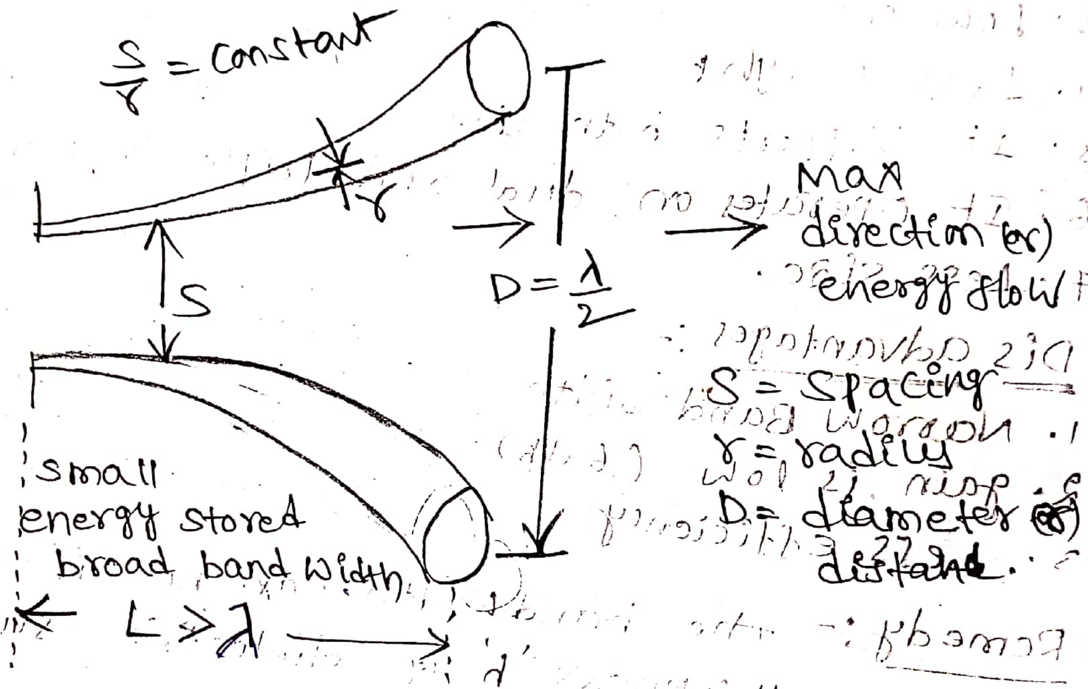
* By increasing Inductance
* Adding reactive components to reduce VSWR. (Voltage Standing Wave Ratio)

Features (Applications)

1. Micro strip antennas are used in mobile & satellite communications
2. It also used in Radio broadcastings.
3. Missile communications.
4. Space craft applications, Radar Communications

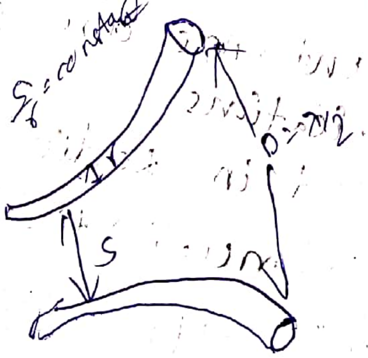
Broad band Antennas:-

* The broad band antenna is the antenna, which is a low-Quality factor radiator, the input impedance is constant over a wide range of frequencies.
* It consists of broad band width. (More b.w)



* The arrows in the diagram represents direction and magnitude of energy flow (radiation)

EX:- Helical antennas.



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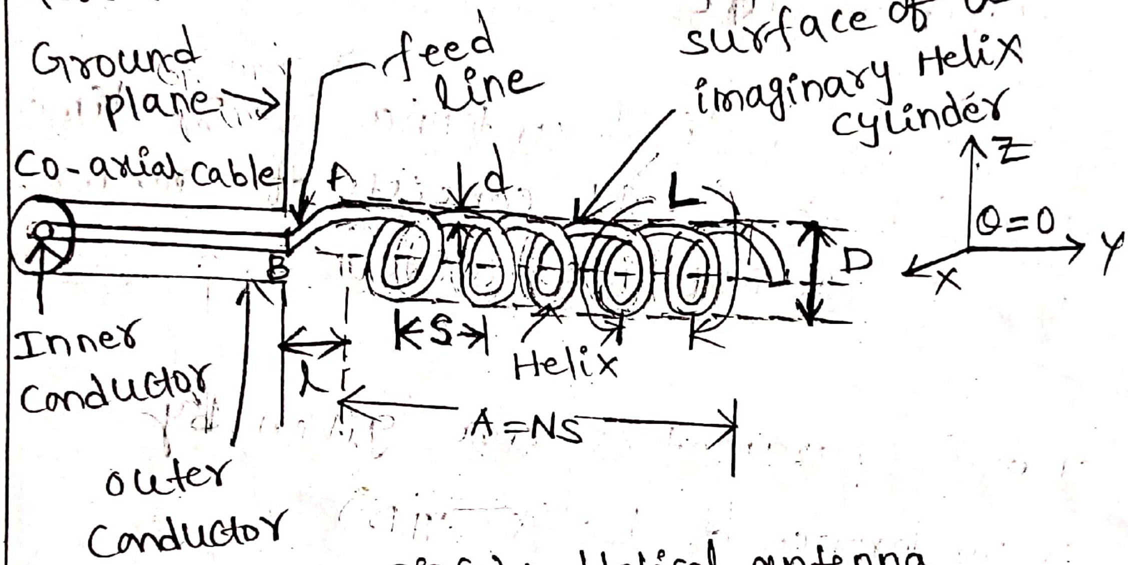
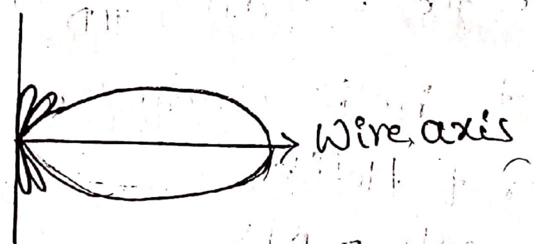
Helical Antenna :- significance :-

Helical antenna is another basic type of radiator It is the simplest antenna to provide circular polarized waves.

* The Helical antenna is broad band VHF and UHF antenna to provide circular polarization characteristics.



fig(a) :- Helical antenna



fig(b):- Radiation pattern.

* The Helical antenna is small dimension in wave length acts as "Guiding structure.
* It consists of helix of thick copper wire is in the shape of screw thread, and is used as an antenna in conjunction with "flat metal plate is called as "ground plate".

* The Helix is generally fed by a Co-axial Cable.
* The inner conductor of cable is connected to one end of the helix, outer conductor is connected to the ground plane.
* finally the mode of radiation depends on the geometric parameters ~~are~~ D and S.

<u>Geometry</u>:- the helical antenna is 3-dimensional geometry form. It consists of geometric shapes of straight line, circle and cylinder shapes.

* The geometric parameters are given by

C = circumference of helix (πD)

α = pitch angle $= \tan^{-1}\left(\frac{S}{\pi D}\right)$

d = diameter of helix Conductor

D = diameter of Helix

A = Axial length $= NS$

N = no. of Turns

S = Turn spacing

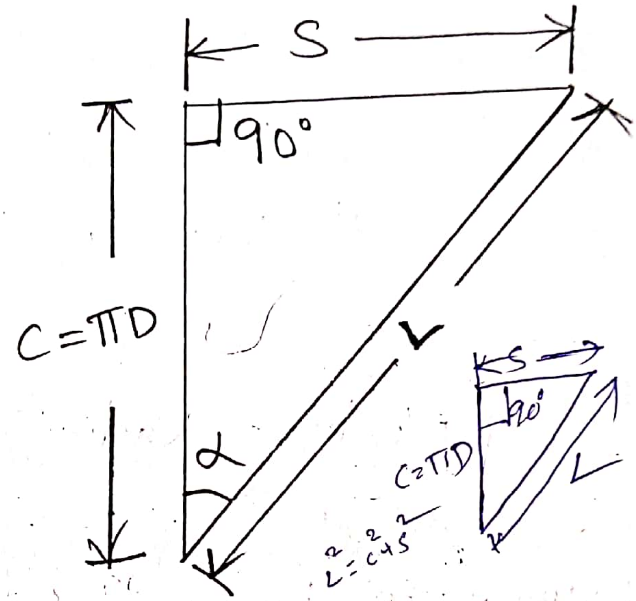
L = Turn Length

l = spacing of helix from ground plane

$C = \pi D$
α = pitch angle
$\tan^{-1}\left(\frac{S}{\pi D}\right)$

* for 'N' no. of turns the total length of the antenna is NS.

Let us consider one turn of helix, the relation between circumference (c), spacing (s), Turn length (L), pitch angle (α) is given by Triangle.



According to pythogorous theorem

$$L^2 = S^2 + C^2$$

$$\Rightarrow L^2 = S^2 + (\pi D)^2$$

$$\therefore \boxed{L = \sqrt{S^2 + (\pi D)^2}}$$

The pitch angle is angle between line parallel to helix wire, and plane perpendicular to helix axis.

$$L^2 = S^2 + \pi^2 D^2$$

$$\boxed{L = \sqrt{S^2 + \pi^2 D^2}}$$

$\therefore$ The pitch angle is 'α'

$$\tan\alpha = \frac{S}{C} = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{S}{C}\right)$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{S}{\pi D}\right)} \quad (\because C = \pi D)$$

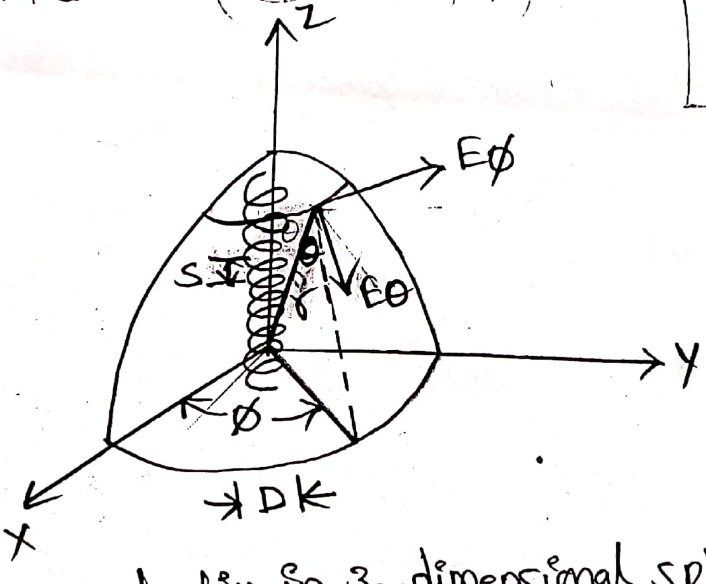
Design considerations for monofilar helical antenna:-

> The helical ~~ang~~ antenna may be radiate in two modes of radiation.

They are (i) Normal mode (or) perpendicular mode
(ii) Axial mode (or) End fire mode (or) beam mode

i) Normal mode of Radiation:-

* In the normal mode of radiation, the radiation field is maximum in broad side way.
* That is, the direction of maximum radiation is perpendicular to helix axis and is circularly polarized waves.
* This mode of radiation is obtained, if the dimensions of the helix is small compared with wavelength λ. $(NL \ll \lambda)$
* Therefore the bandwidth of a small helix is very narrow and radiation efficiency is low.



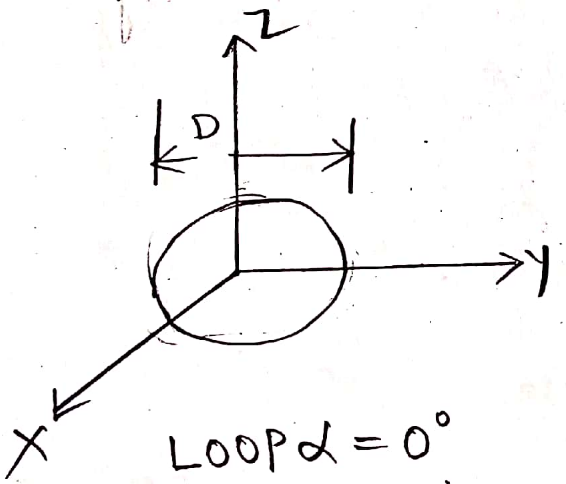
helix in 3-dimensional spherical coordinates

The bandwidth and radiation efficiency can be increased by increasing the size of helix and to have current in phase. (16)

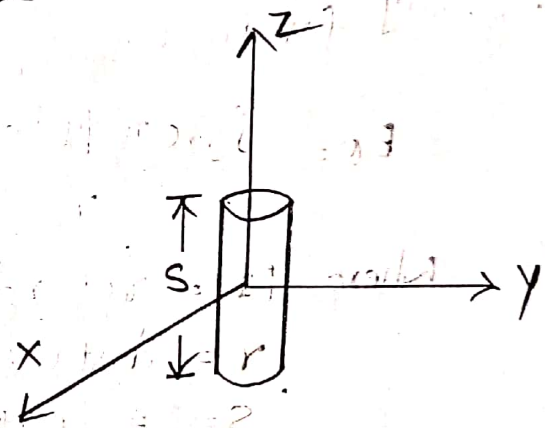
* The radiation pattern is a combination of equivalent radiation from a short dipole located on the same helix axis and a small loop which is also coincidents (or) co-axial with helix axis.

* In a helix if spacing $S \to 0$, diameter is fixed and pitch angle $\alpha = 0°$ then helix becomes a loop.

* if S = constant and Diameter $D \to 0$ and pitch angle $\alpha = 90°$ then helix becomes a short dipole.



LOOP $\alpha = 0°$
D = fixed (or) constant



Short dipole
$\alpha = 90°$,
S = constant

* The polarizations are at right angle and the phase angles are 90° at any point in space.

* Hence the resultant field is either circularly polarized (or) elliptically polarized depend upon field strength ratio.

A helix antenna may be considered of having a number of small loops and short dipoles connected in series in which loop diameter is same as helix diameter and helix spacing 's' is same as dipole length (L).

The far field of small loop is given by

$$\boxed{E_\phi = \frac{120\pi^2 |I| \sin\theta}{r} \frac{A}{\lambda^2}} \rightarrow (1)$$

$|I|$ = retarded current

r = distance

A = Area of loop = $\frac{\pi D^2}{4}$

similarly far field of a short dipole is given by

$$E_\theta = \frac{j\,60\pi |I| \sin\theta}{r} \frac{L}{\lambda}$$

where $|I|$ = retarded current

r = distance

$S = L$ = Length of dipole

$$\therefore \boxed{E_\theta = j\,\frac{60\pi |I| \sin\theta}{r} \frac{S}{\lambda}} \rightarrow (2)$$

The axial ratio (AR) of Elliptical polarization is

$$AR = \frac{|E_\theta|}{|E_\phi|} = \frac{\left| j\,\frac{\cancel{60\pi} \cancel{|I|\sin\theta}}{\cancel{r}} \cdot \frac{S}{\cancel{\lambda}} \right|}{\left| \frac{\cancel{120}^{2}\pi^{\cancel{2}} \cancel{|I|\sin\theta}}{\cancel{r}} \cdot \frac{A}{\lambda^{\cancel{2}}} \right|} = \frac{S}{\left(2\pi \frac{A}{\lambda}\right)}$$

$$\Rightarrow A.R = \frac{S\lambda}{\cancel{2}\pi \cdot \frac{\pi D^2}{\cancel{4}_2}} = \frac{2S\lambda}{\pi^2 D^2} \qquad \left(\because A = \frac{\pi D^2}{4}\right)$$

$$\therefore \boxed{A.R = \frac{2S\lambda}{\pi^2 D^2}} \rightarrow (3) \text{ Axial Ratio.}$$

* When Axial Ratio is '0' the elliptical polarization becomes Linear horizontal polarization.

* When Axial Ratio (AR) is '∞' the elliptical polarization becomes Linear Vertical polarization.

* When Axial Ratio is '1' (unity) the elliptical polarization becomes circular polarization.

$$AR = 1 = \frac{|E_\theta|}{|E_\phi|} \text{ (or) } |E_\theta| = |E_\phi|$$

$$\Rightarrow AR = 1 = \frac{2S\lambda}{\pi^2 D^2}$$

$$\Rightarrow 2S\lambda = \pi^2 D^2$$

$$\therefore \boxed{S = \frac{\pi^2 D^2}{2\lambda}} \rightarrow (4)$$

$$\boxed{S = \frac{C^2}{2\lambda}} \rightarrow (5)$$

($\because$ Circumference $C = \pi D$)

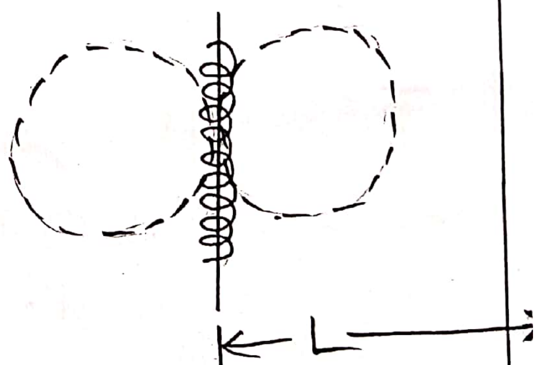
$\therefore$ The pitch angle is given by

$$\alpha = \tan^{-1}\left(\frac{S}{\pi D}\right) = \tan^{-1}\left(\frac{\frac{\cancel{\pi^2} \cancel{D^2}}{2\lambda}}{\cancel{\pi D}}\right)$$

$$\Rightarrow \boxed{\alpha = \tan^{-1}\left(\frac{\pi D}{2\lambda}\right)}$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{C}{2\lambda}\right)}$$

This is the pitch angle to get circular polarization.

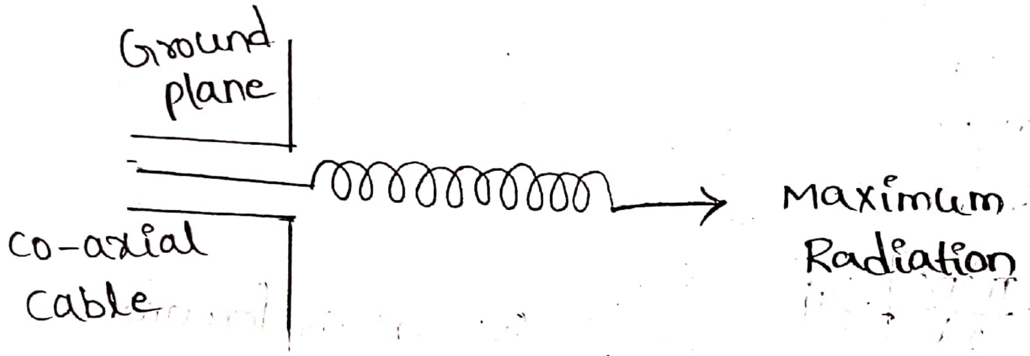


Radiation pattern for Normal mode of Radiation

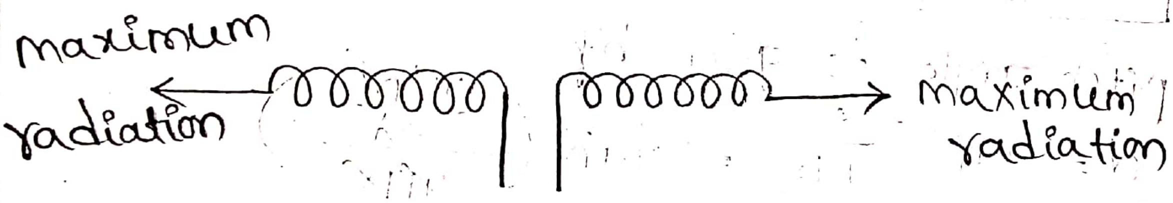
$\therefore$ practically this mode is not suitable and hardly used.

Axial (or) Beam mode of Radiation :-

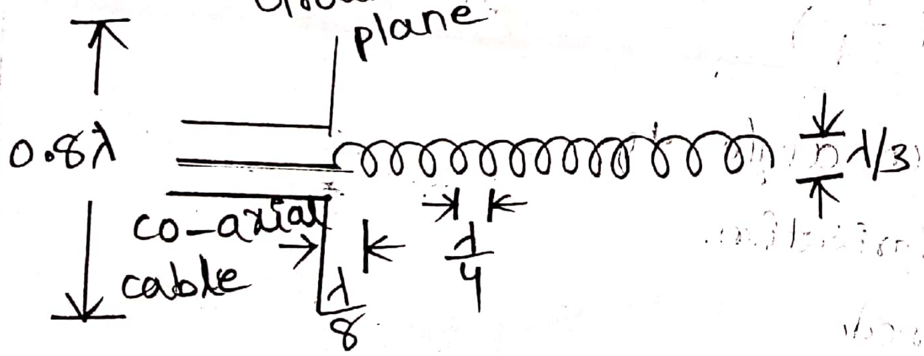
* In axial mode of radiation, the radiation field is maximum in end-fire direction.
* That is the direction of maximum radiation is co-incidence with Helix axis, the polarization is circular (or) nearly circular
* This mode occurs when the helix circumference (C) and spacing (S) are in the order of one wavelength.



(a) With Co-axial cable



(b) Two wire Transmission line



(c) Typical dimensions

* This mode produces a broad and fairly directional beam in the axial direction with minor lobes at oblique angles.
* This mode of radiation is used in most practical applications.
* The axial mode of radiation is produced very easily by raising helix circumference (C) of the order of one wavelength (λ) and spacing (S) is $\frac{\lambda}{4}$.
* The ground plane having atleast half wavelength in diameter.
* The pitch angle 'α' varies from 12° to 18° and optimum pitch angle is 14°.
* The terminal impedance is $100\,\Omega$ resistive at frequency $C \cong \lambda$ $(\because \lambda = \frac{c}{f})$
* Generally in axial mode, the terminal impedance of helical ~~and~~ antenna lies between $100\,\Omega$ to $200\,\Omega$
  $\therefore$ for $\pm 20\%$ approximation, the terminal impedance is given by $\boxed{R = \frac{140\,C}{\lambda}}$ ohms.
* The antenna gain and beam width depends on the helix axial length $(A = NS)$

The half power beam width is given by

$$\boxed{(HPBW)\ \theta_{(-3db)} = \frac{52^\circ}{C}\sqrt{\frac{\lambda^3}{NS}}\ \text{degrees}}$$

λ = free space wavelength
C = circumference ~~wavelength~~
N = no. of Turns
S = spacing

The beamwidth between first nulls is

$$BWFN = 2 \times HPBW$$

$$\boxed{BWFN = \frac{104^\circ}{C}\sqrt{\frac{\lambda^3}{NS}}\ \text{degrees}}$$

The maximum directive gain (or) directivity is

$$\boxed{D = \frac{15NSC^2}{\lambda^3}}$$

Axial Ratio $\boxed{AR = 1 + \frac{1}{2N}}$

Farfield pattern is given by

$$\boxed{\begin{aligned} E &= \sin\left(\frac{\pi}{2N}\right)\cos\theta \cdot \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\frac{\psi}{2}} \\ \psi &= 2\pi\left[\frac{S}{\lambda}(1-\cos\theta) + \frac{1}{2N}\right] \end{aligned}}$$

$\alpha = 12^\circ$ to 15°, $N \geq 3$, $NS \leq 10$, $C = \frac{3}{4}\lambda$ to $\frac{4}{3}\lambda$

Features of Helical Antenna:-

(19)

* Helical antenna is used for circular polarization
* The helical antenna is used most widely in VHF and UHF bands.
* The axial mode of helical antenna is most widely used.
* The antenna in axial mode has larger band width
* It's construction is simple and directivity is higher.

Applications of Helical antennas:-

* The dimensions for axial mode are not critical.
* Hence helical antennas are used to achieve circularly polarized waves over wide band width.
* A single helical antenna (or) array of helical antennas are useful in Transmitting (or) receiving VHF signals through the Ionosphere.
* The helical are also most useful in satellite communications.
* these antennas are able to produce circular polarized waves.

A. Narasimha Reddy,
ECE Dept.

UNIT-5 :- VHF, UHF, and Microwave Antennas :-

OM SRI Sai Ram

Introduction to Reflector antennas :-

→ The reflector antennas are most important in microwave radiation applications. At microwave frequencies the physical size of the high gain antenna becomes so small to produce desired directivity.

→ In reflector antenna another antenna is required to excite it.

→ The antennas such as dipole, Horn, slot which feeds the reflector antenna.

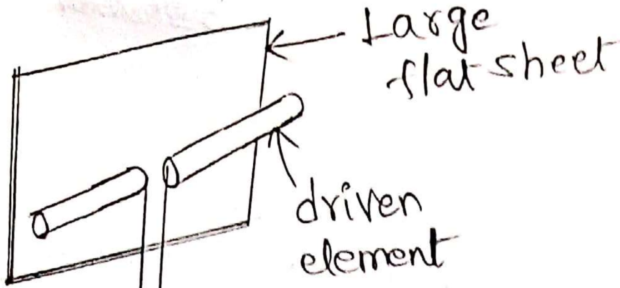
→ Dipole, Horn, slot antenna is called as "primary antenna", and reflector is called as "secondary antenna."

→ Reflector antenna canbe represented in any geographical configuration, but the most commonly used shapes are plane reflector, corner reflector and curved (or) parabolic reflectors.

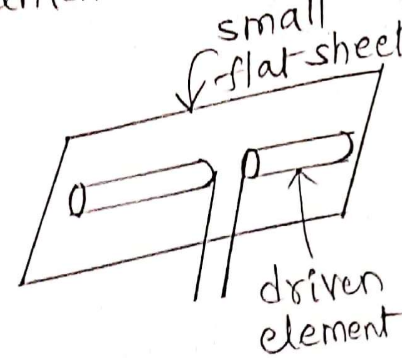
→ By using reflectors, the backward radiations from the antenna can be eliminated. Thus improving radiation pattern of an antenna.

→ using reflectors, the radiation pattern of a radiating antenna can be modified.

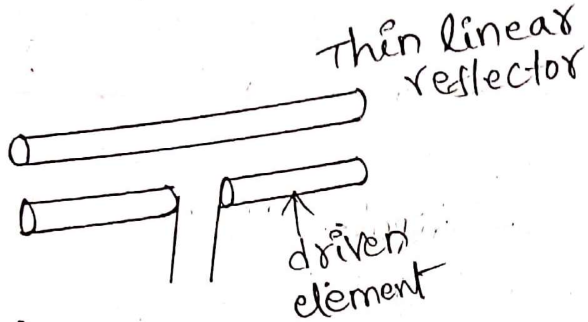
Different types of reflector antennas are



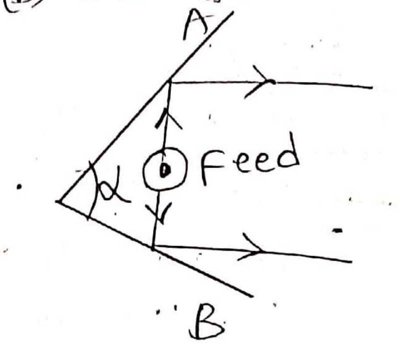
(a) Large flat sheet



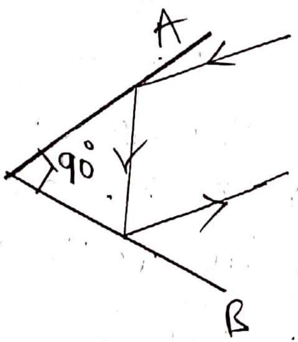
(b) small flat sheet



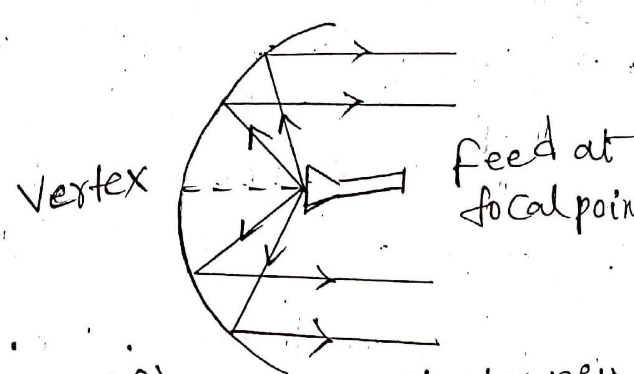
(c) Thin linear reflector antenna



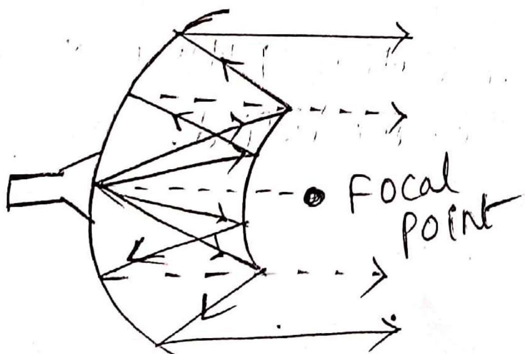
(d) Active corner reflector.



(e) passive corner reflector.

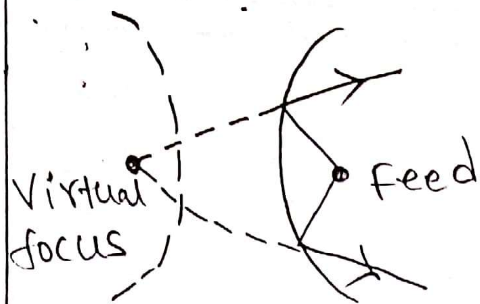


(f) curved reflector with front feed.

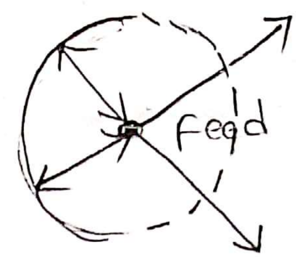


(h) elliptical reflector

(g) curved (or) parabolic reflector with Cassegrain feed.



(i) hyperbolic reflector.

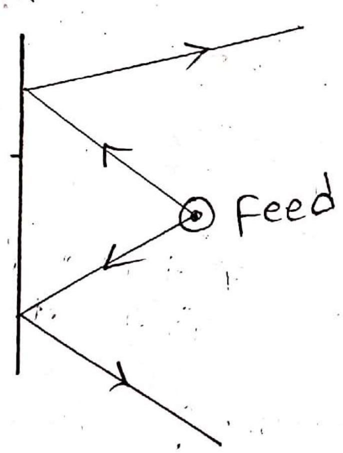


(j) Circular reflector

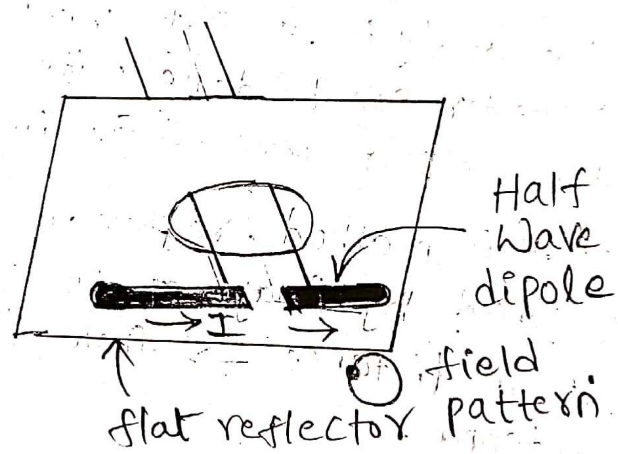
Flatsheet (or) plane reflectors :-

* The plane reflector is the simplest form of the reflector antenna. A flat sheet reflector can be considered to be made up of two flat sheets intersecting each other at an angle $\alpha = 180°$

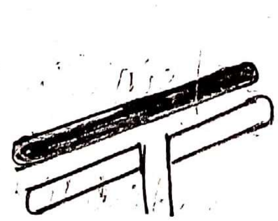
* When the plane reflector is placed infront of the feed, the energy is radiated in the desired directions.



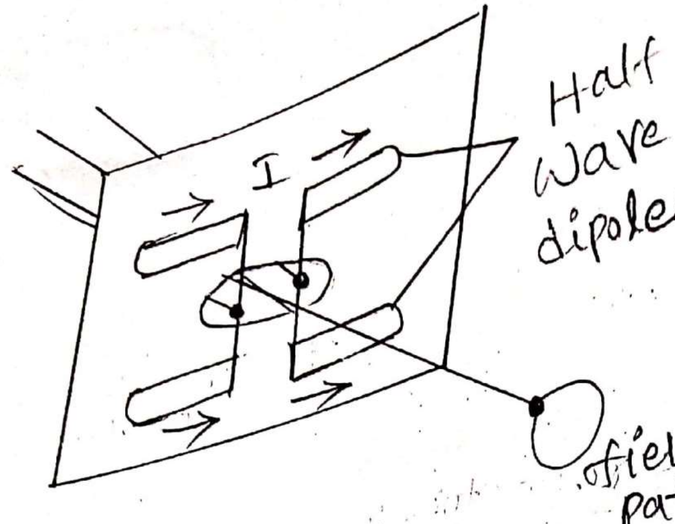
(a) plane reflector.



Examples :- Half wave dipole with plane reflector.



half wave dipole with reflector element

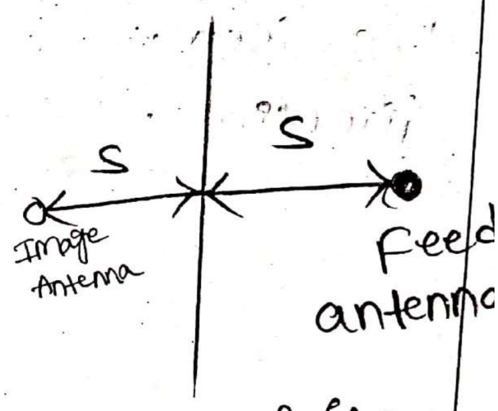


Half wave dipole array with plane reflector.

* The polarization of the radiating source and its position with respect to the reflector both are important as one can control radiating pattern, directivity, Impedance.

* The analysis of flat sheet reflector can be done with the help of method of images.

* In this method, reflector can be replaced by image of an antenna at a distance 2S from feed antenna.



Antenna & its image at a distance 's'.

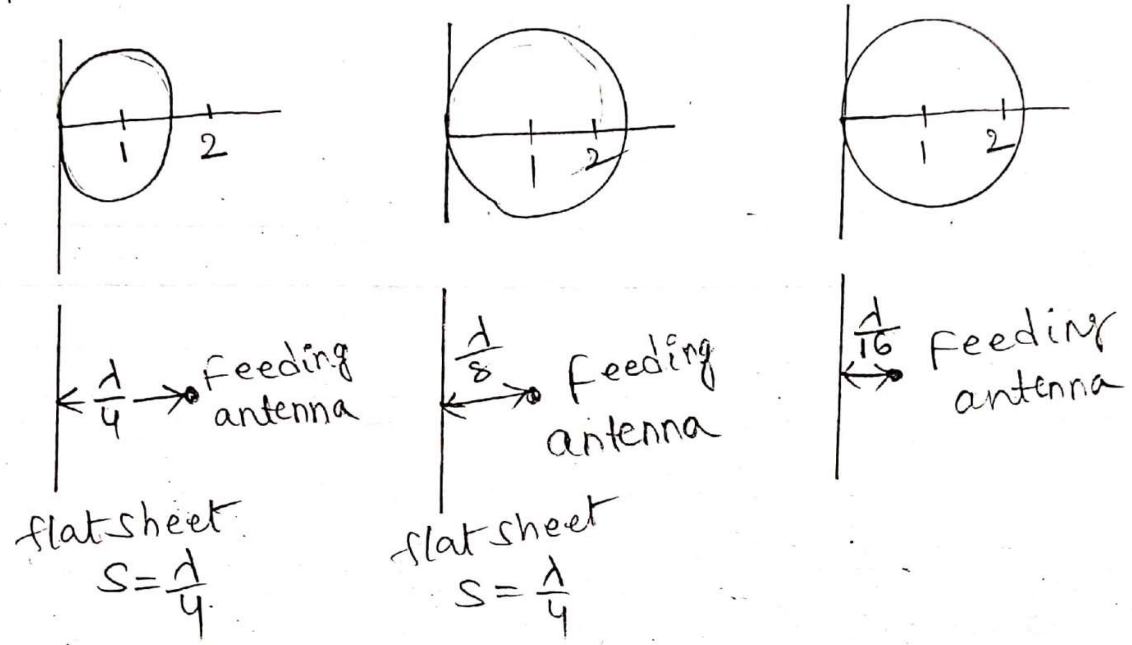
For an infinite plane reflector, assuming zero reflector losses, the gain of a $\frac{\lambda}{2}$ dipole antenna at a distance 's' is given by

$$G_f(\phi) = 2\sqrt{\frac{R_{11} + R_{LOSS}}{R_{11} + R_{LOSS} - R_{12}}}\,\left|\sin(S_r \cos\phi)\right|$$

and $S_r = \left(\frac{2\pi}{\lambda}\right) S$. ($\because S_r$ = radial distance, S = distance.)

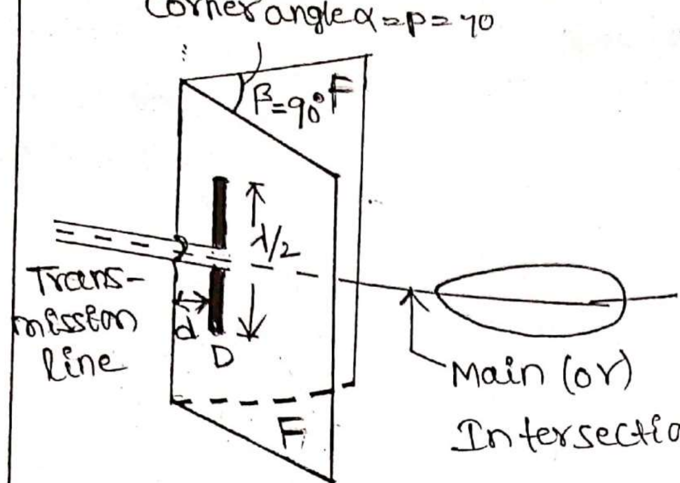
The gain of reflector relative to half wave dipole antenna is a function of the spacing between flat sheet and half wave dipole antenna.

* When the spacing between half wave dipole and infinite sheet decreases, the gain will be increases.



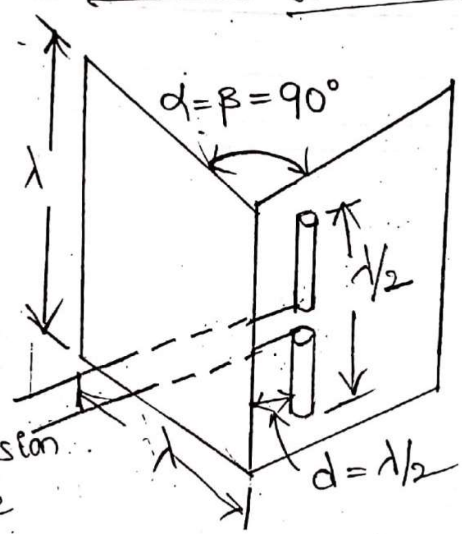
Corner Reflector :-

* The corner reflector antenna can be considered to be made up of two flat sheets meet at angle $\alpha=90°$.
* The flat reflecting sheets meeting at angle (or) corner form an effective directional antenna.
* The corner reflector antenna is a driven antenna associated with a reflector
* Generally driven antenna is a Half wave dipole and reflector can be constructed of two flat sheets meet at a corner (or) angle to form corner.
* This arrangement with corner reflector and driven antenna is known as "corner reflector antenna".

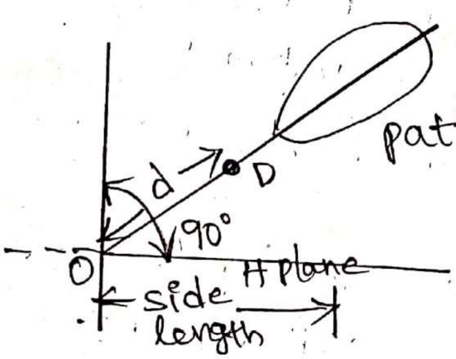


f = flat reflecting sheets
D = driven antenna

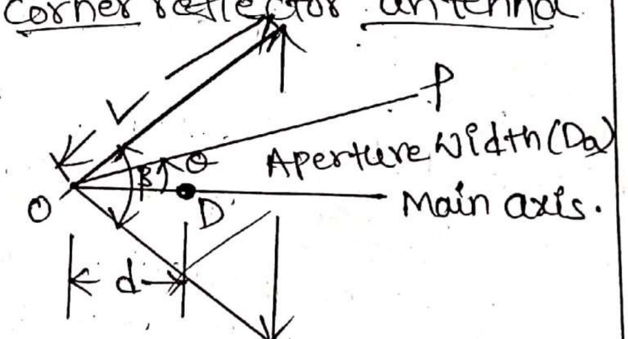
(a) Vertical corner reflector antenna.



(b) Horizontal corner reflector antenna

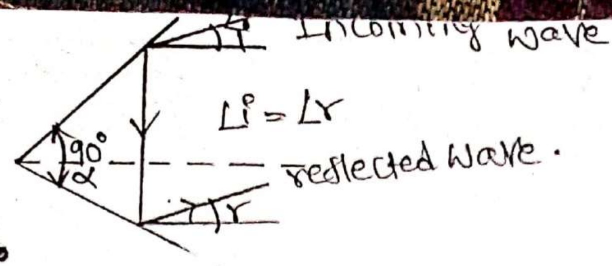


(c) Radiation pattern.



d = spacing between driven elements
α = β = corner angle
D = driven antenna.

(d) Active corner reflector



(e) passive corner reflector.

* If corner angle $\beta = \alpha = 90°$ then the two flat sheets meeting at a right angle forming a square corner reflector.
* When the corner reflector with the driven antenna is called "active corner reflector" for a wide range of corner $0 < \beta < \pi$
* When the corner reflector without the driven antenna is called " passive corner reflector" for a wide range of angle of incidence $0 < i < \pm \frac{\pi}{4}$
* The corner reflector antenna may be analysed by using the method of images for corner angle.

$\boxed{\alpha = \beta = \frac{180°}{n}}$ Where n = an integer $= 1, 2, 3 \ldots$

thus if $n = 1$, $\beta = 180°$ (or) π radian $\longrightarrow$ flat sheet reflector

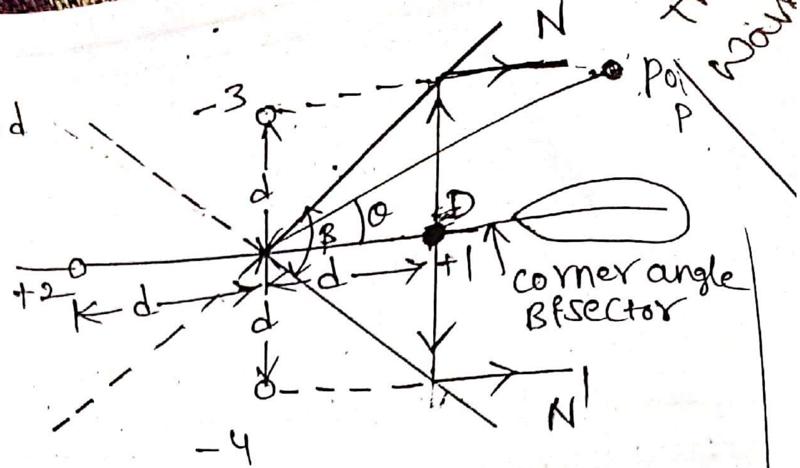
if $n = 2$, $\beta = 90°$ (or) $\frac{\pi}{2}$ radian $\longrightarrow$ square corner reflector.

if $n = 3$, $\beta = 60°$ (or) $\frac{\pi}{3}$ radian $\rightarrow$ corner reflector $60°$

if $n = 4$ $\beta = 45°$ (or) $\frac{\pi}{4}$ radian $\rightarrow$ corner reflector $45°$

$\therefore$ By method of images corner angles of $\pi, \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}$ can only be used.

* Let us consider method of images for square corner reflector



Square corner reflector with driven element (+1) and three images (+2, -3, -4).

* The driven antenna is shown by 'D' and three images (+2, -3, -4) corresponding to driven antenna (+1).
* The driven antenna (half wave dipole) and its three images carry equal currents.
* driven antenna (+1) and image element (+2) are in same phase & -3 and -4 image elements are also in same phase.
* But there exists a 180° phase shift between phase of elements (+1, +2) and (-3, -4). The two negative images corresponds to single reflection of rays N and N', third +ve image (+2) corresponds to driven element (+1)

The field pattern $E_\phi(\theta)$ in the horizontal plane at a large distance r from the antenna is given by

$$E_\phi(\theta) = K' I_1 [\cos(\beta d \cos\theta) - \cos(\beta d \sin\theta)] \rightarrow (1)$$

Where K' = Constant,
I_1 = current in each element
$\beta = \frac{2\pi}{\lambda}$
d = distance between driven element & corner along bisector

the terminal voltage at the centre of the half wave dipole can be expressed as

$$V_1 = I_1 Z_{11} + I_1 Z_{12} - I_1 Z_{13} - I_1 Z_{14} \rightarrow (2)$$

$$V_1 = I_1 (Z_{11} + Z_{12} - 2Z_{14}) \rightarrow (3) \quad (\because Z_{13} = Z_{14})$$

Where Z_{11} = self impedance of driven antenna (+1) $= 73\Omega$

Z_{12} = mutual impedance between +1 and +2

Z_{13} = Mutual impedance between +1, −3.

Z_{14} = Mutual impedance between +1, −4.

the power supplied to driven antenna is

$$P = I_1^2 R$$

$$\Rightarrow I_1^2 = \frac{P}{R} \Rightarrow \boxed{I_1 = \sqrt{\frac{P}{R}}} \rightarrow (4)$$

from eqn (3)

$$\frac{V_1}{I_1} = Z = Z_{11} + Z_{12} - 2Z_{14} \quad \text{(OR)}$$

$$\frac{V_1}{I_1} = R = R_{11} + R_{12} - 2R_{14} \rightarrow (5)$$

from equations (4), (5)

$$I_1 = \sqrt{\frac{P}{R}} = \sqrt{\frac{P}{R_{11} + R_{12} - 2R_{14}}} \rightarrow (6)$$

substitute eq (6) in eq (1)

$$E_\phi(0) = K' \sqrt{\frac{P}{R_{11} + R_{12} - 2R_{14}}} \times [\cos(\beta d \cos\theta) - \cos(\beta d \sin\theta)]$$

if reflector is removed then $R_{12} = R_{14} = 0$ then

$$\boxed{E_\phi(0)_{\lambda/2} = K' \sqrt{\frac{P}{R_{11}}}}$$

The gain in the θ direction is given by

$$G = \frac{E_\phi(\theta)}{E_\phi(\theta)_{\lambda/2}}$$

$$\Rightarrow G = \frac{\cancel{K'}\sqrt{\frac{\cancel{P}}{R_{11}+R_{12}-2R_{14}}} \times \left[\cos[\beta d\cos\theta] - \cos[\beta d\sin\theta]\right]}{\cancel{K'}\sqrt{\frac{\cancel{P}}{R_{11}}}}$$

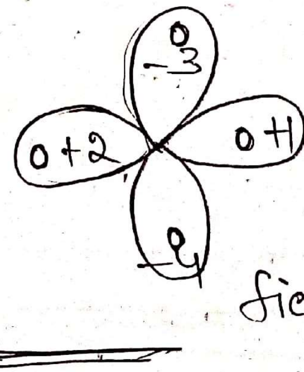
$$\Rightarrow \boxed{G = \sqrt{\frac{R_{11}}{R_{11}+R_{12}-2R_{14}}} \times \left[\cos(\beta d\cos\theta) - \cos(\beta d\sin\theta)\right]} \rightarrow \text{eq } 7$$

Where $[\cos(\beta d\cos\theta) - \cos(\beta d\sin\theta)]$ = pattern factor

$$\sqrt{\frac{R_{11}}{R_{11}+R_{12}-2R_{14}}} = \text{coupling factor.}$$

The maximum radiation from the corner reflector antenna is in the direction $\theta = 0$ hence putting $\theta = 0$ in eq 7

$$\therefore \boxed{G_0 = \sqrt{\frac{R_{11}}{R_{11}+R_{12}-2R_{14}}}\,[\cos(\beta d) - 1]} \rightarrow 8$$



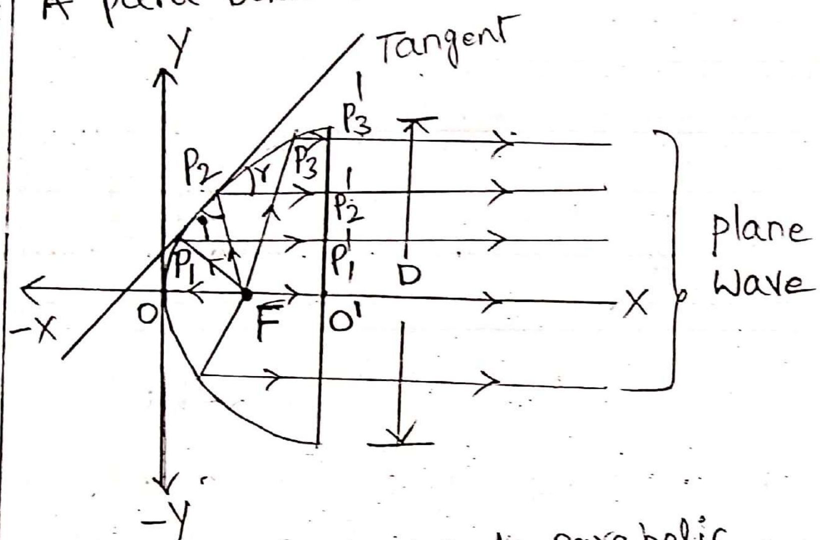
($\because$ Combination of broadside & end fire patterns in UNIT-3)

field pattern.

parabolic reflectors :- (2-Dimensional)

A parabola may be defined as the focus of a point which moves in such a way that its distance from the fixed point (focus) plus its distance from a straight line (directrix) is constant.

* A para bola is a two dimensional plane curve.



OF = focal length = f

K = constant depends on shape of parabola curve

F = focus

O = Vertex

OO' = Axis of parabola

Geometry of parabolic Reflector

By definition of parabola

$$FP_1 + P_1P_1' = FP_2 + P_2P_2' = FP_3 + P_3P_3' = \text{constant} = K \text{ (say)}$$

the equation of parabola curve interms of its coordinate is given by

$$\boxed{y^2 = 4fx}$$

* the open mouth (D) of the parabola is known as aperture.
* Generally F/D ratio is an important parameter of parabolic reflector. its value is 0.25 to 0.50.

* The parabola converts a spherical wave front coming from a focus into a plane wave front at the mouth (D) of the parabola. If a . perper . of

* Let us consider a source of radiation at the focus. A ray starts from focus (F) with respect to parabolic axis (OO')

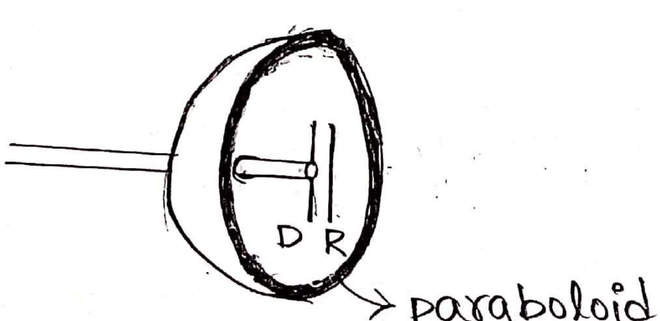
* Let a tangent is drawn at P_2 on the curve. According to law of reflection the angle of incidence $\angle i$ and angle of reflection ($\angle r$) will be equal.

* this results the reflected ray is parallel to the parabolic axis. That means all the waves originating from the focus will be reflected parallel to the parabolic axis.

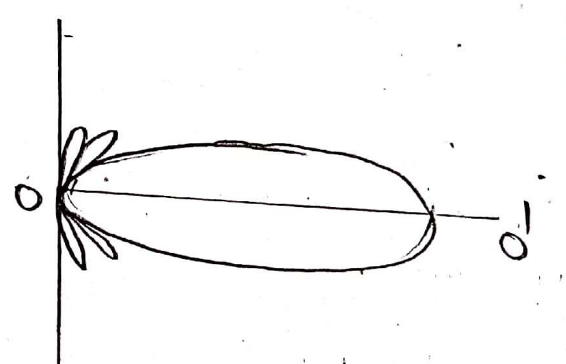
Paraboloidal Reflector (or) Micro wave Dish (3-Dimensional)

* A practical reflector is a three dimensional curved surface. Therefore a practical reflector is formed by rotating a parabola about its axis (OO'). The generated surface is known as "paraboloid". (or) Micro wave dish.

* paraboloid produces a parallel beam of circular cross section, because the mouth of the paraboloid is circular



D= Dipole
R= Reflector



Radiation pattern of paraboloid of

If a third cartesian coordinate z has its axis perpendicular to both x-axis and y-axis then eqn of paraboloid will be

$$\boxed{y^2 + z^2 = 4fx}$$

* the intersection of any plane perpendicular to x-axis with the paraboloid surface is a circle.

Characteristics :-

If the feed or primary antenna is isotropic then the paraboloid will produce a beam of radiation. Assume the circular aperture is large, the beam width between first nulls is given by

$$\boxed{BWFN = \frac{140\lambda}{D} \text{ degree}}$$

Where
λ = free space wave length
D = diameter of aperture in 'm' (or) Mouth diameter.

The Beam width between first nulls for large uniformly illuminated rectangular aperture is given by

$$\boxed{BWFN = \frac{115\lambda}{L} \text{ (degree)}}$$

Where L = length of aperture in λ

Half power Beam Width for large circular aperture is given by

$$\boxed{HPBW = \frac{58\lambda}{D} \text{ degree}}$$

The directivity D of a large uniformly illuminated aperture is

$$\boxed{D = \frac{4\pi A}{\lambda^2}}$$

For a circular aperture

$$D = \frac{4\pi}{\lambda^2}\left(\frac{\pi D^2}{4}\right) = \frac{\cancel{4}\pi}{\lambda^2} \times \frac{\pi D^2}{\cancel{4}} \qquad \left(\because A = \frac{\pi D^2}{4} \text{ for circle}\right)$$

$$\therefore D = \frac{\pi^2 D^2}{\lambda^2} = \pi^2\left(\frac{D}{\lambda}\right)^2$$

Directivity: $\boxed{D = 9.87\left(\frac{D}{\lambda}\right)^2}$

where D = diameter of aperture in 'λ'

We know that ~~Actual~~ capture area $A_0 = KA$

where A_0 = capture area

A = Actual area of mouth

K = constant depends on type of antenna used for feed

= 0.65 (approx for dipole)

Therefore power gain of circular Aperture paraboloid with respect to half wave dipole is given by

$$G_p = \frac{4\pi A_0}{\lambda^2} = \frac{4\pi \times KA}{\lambda^2} \qquad (\because A_0 = KA)$$

$$= \frac{\cancel{4}\pi K}{\lambda^2}\left(\frac{\pi D^2}{\cancel{4}}\right) \qquad \left(\because A = \frac{\pi D^2}{4} \text{ for circular aperture}\right)$$

$$G_p = \pi^2 K \cdot \frac{D^2}{\lambda^2}$$

$$\therefore G_p = (3.14)^2(0.65)\left(\frac{D}{\lambda}\right)^2 \qquad (\because K = 0.65)$$

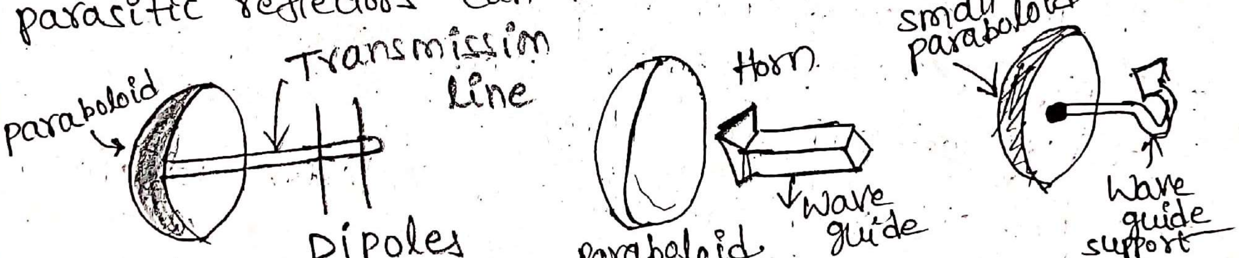
$$G_p = 6.389\left(\frac{D}{\lambda}\right)^2$$

$$\boxed{G_p \cong 6\left(\frac{D}{\lambda}\right)^2}$$

Types of feeds :-

Q2

* A parabolic reflector antenna as a system consists two parts.
  - → source
  - → Reflector
* The source placed at the focus is called "primary" radiator, while the reflector is called "secondary" radiator.
* The primary radiator (or) the source is commonly called "feed radiator" (or) simply feed.
* The simplest type of the feed that can be used is a dipole antenna. But it is not suitable feed for the parabolic reflector antenna.
* Instead of only dipole, a feed consisting dipole with parasitic reflectors can be used as feed system.



(a) dipole endfire feed.

* The most widely used feed system in the parabolic reflector antenna is horn antenna. Horn antenna is feed with a wave guide.

There are two types of feeds :

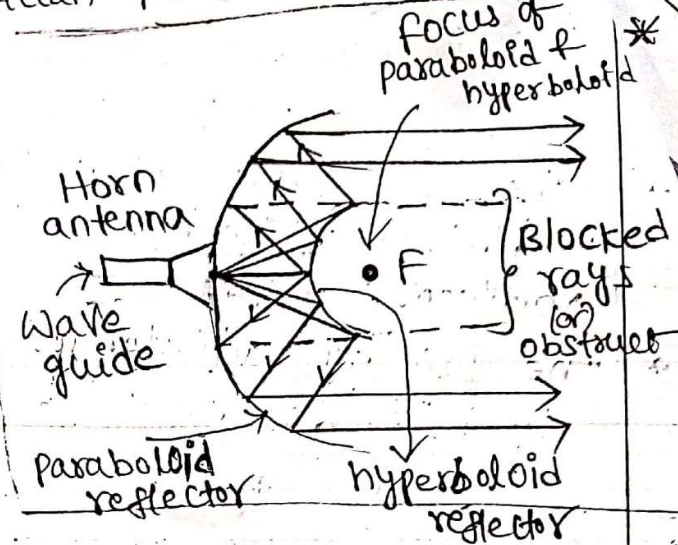
→ cassegrain feed system

→ offset feed system.

Cassegrain feed system :-

* This system of feeding paraboloid reflector is named after a mathematician prof. Cassegrain discovered.

* In all the feed systems the feed is located at the focus. But in cassegrain feed, the feed radiator is placed at the vertex of parabolic reflector.



* This system uses a hyperboloid reflector placed such that one of the foci co-incides with the focus of parabolic reflector, (or) paraboloid.

* This hyperboloid reflector is called "Cassegrain" secondary (or) sub-reflector.

* The primary radiator (or) feed radiator generally used a horn antenna

Advantages :-

→ It reduces spill over and minor lobe radiations.

→ The system has ability to place a feed at convenient place.

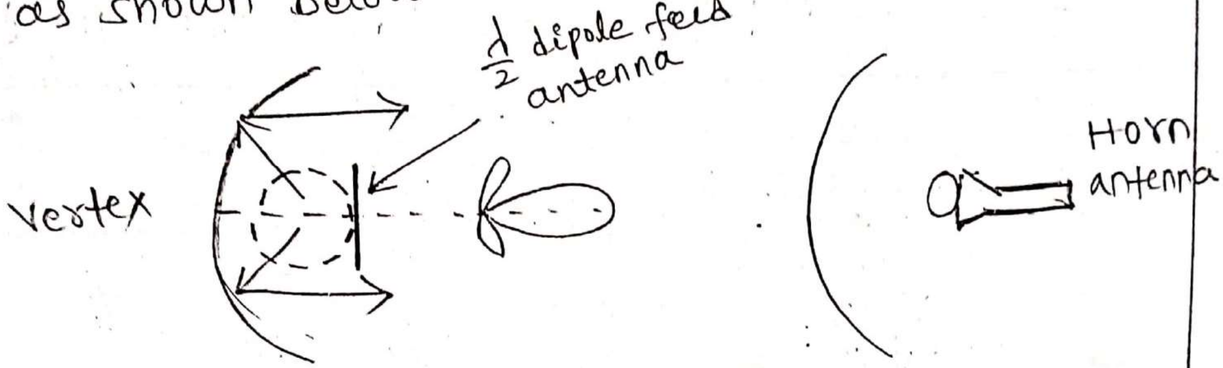
Dis-adv :-

→ There is a region of blocked rays infront of cassegrain reflector, that means Aperture Blocking.

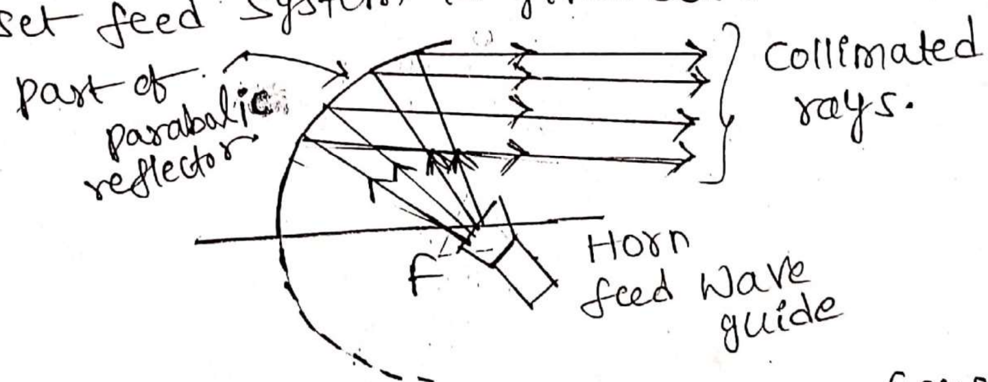
Offset feed system:-

To overcome the aperture blocking effect in cassegrain feed, we are using the offset feed system.

* By suitably selecting primary antenna, correct directional pattern for any arrangement can be obtained.
* The parabolic reflector can be fed using $\frac{\lambda}{2}$ antenna with a small ground plane (or) a horn antenna as shown below.



* The offset feed system is given below.



* Here the feed radiator is placed at the focus. With this system all the rays are properly collimated without formation of the region of blocked rays.

* Therefore the aperture blocking effect can be reduced by offset feed system.

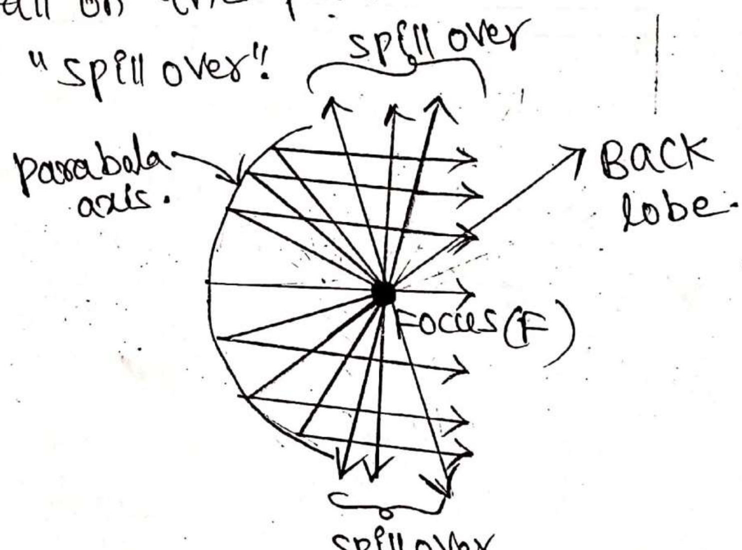
F/D Ratio :-

* In the case of paraboloids, the ratio of focal length to dish diameter is referred as the F/D ratio.

$$\frac{F}{D} = \frac{\text{Focal length}}{\text{Diameter of dish.}}$$

Spillover :- The waves originating from focus will be reflected parallel to the axis of parabola.

* Some of the waves originating from focus may not fall on the parabola. This phenomenon is called "spill over".

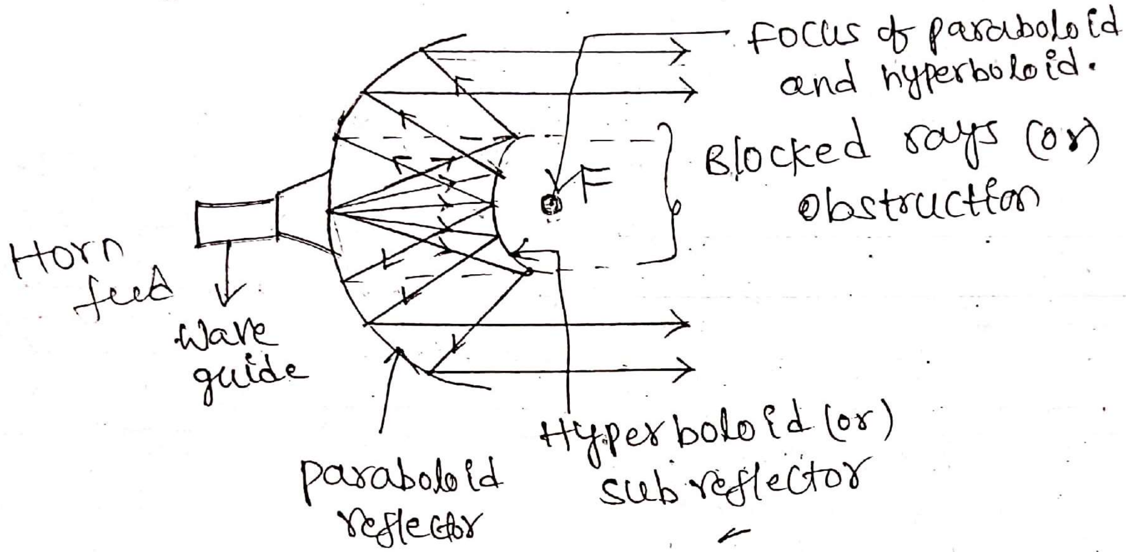


Back lobe :-

* While Receiving spill over, the noise pick-up increases which is some defect. In addition to this, few radiations originated from the primary radiators are observed in forward direction, such radiations get added with desired parallel beam. This is called as "Back lobe radiation".

Aperture Blocking :-

An unwanted phenomenon occured in cassegrain feed Parabolic antennas, in which the obstruction of primary reflector takesplace due to the effect of sub reflector known as "aperture blocking".



Horn Antennas :-

* the horn antenna is most widely used simplest form of the microwave antenna. the horn antenna serves as a feed element for large radio astronomy, communication dishes and satellite tracking over the world.
* the horn antenna can be considered as a flared out (or) opened out, wave guide, which is flared out (or) opened out.
* When one end of the wave guide is feeded and other end is open, it radiates in open space in all directions.
* As compared with the two wire Transmission line, the radiation through the waveguide is larger.

- In waveguide, the small amount of power is ra ated in incident wave, while due to open circuit at other end large amount of power is reflected back.

- As one end of the waveguide is open circuited, the impedance matching with the free space is not perfect

- So at the edges of waveguide, diffraction occurs. that means interference of electromagnetic waves.

- Therefore To overcome these problems the mouth of the waveguide is flared (or) opened out in the shape of Horn.

Types of Horn Antennas :-

- A horn antenna is nothing but a flared out (or) opened out waveguide. The main function is to produce an uniform phase front with a aperture larger than waveguide to give higher directivity

Horn Antennas.

- Rectangular Horn antennas
  - Sectorial Horn antennas
    - E-plane Sectorial Horn
    - H-plane Sectorial Horn.
  - pyramidal Horn antennas
- Circular Horn antennas.
  - Conical Horn
  - Bi-Conical Horn.

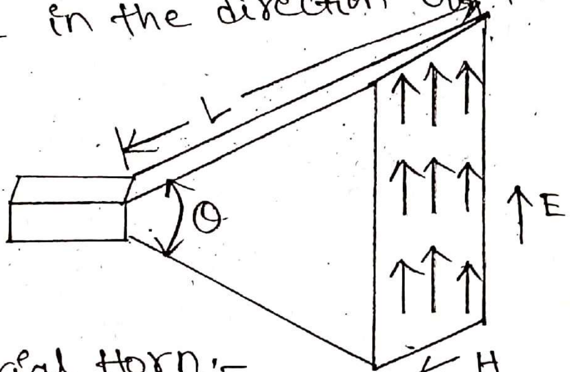
The rectangular Horn antennas are fed with rectangular waveguide, while the circular horn antennas are fed with circular wave guide.

* Depending upon the direction of flaring (opening), the rectangular horns are further classified as sectorial horn and pyramidal horn.

* A sectorial horn is obtained if the flaring is done in one direction only. This is further classified as E-plane sectorial horn and H-plane sectorial horn.

<u>E-plane sectorial horn</u> :-

* The E-plane sectorial horn is obtained, when the flaring is done in the direction of the electric field vector.

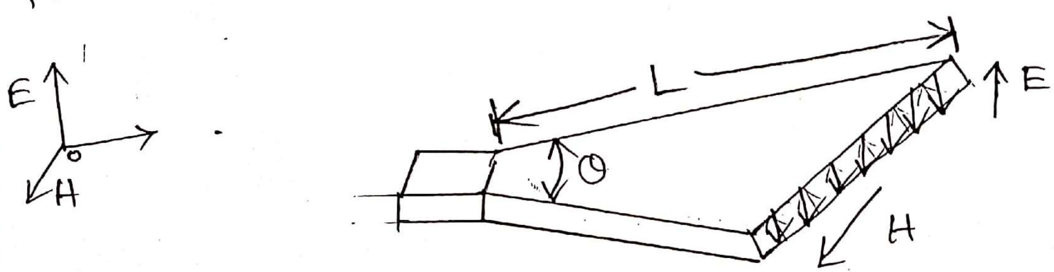


L = Axial length

θ = Half of the flare angle

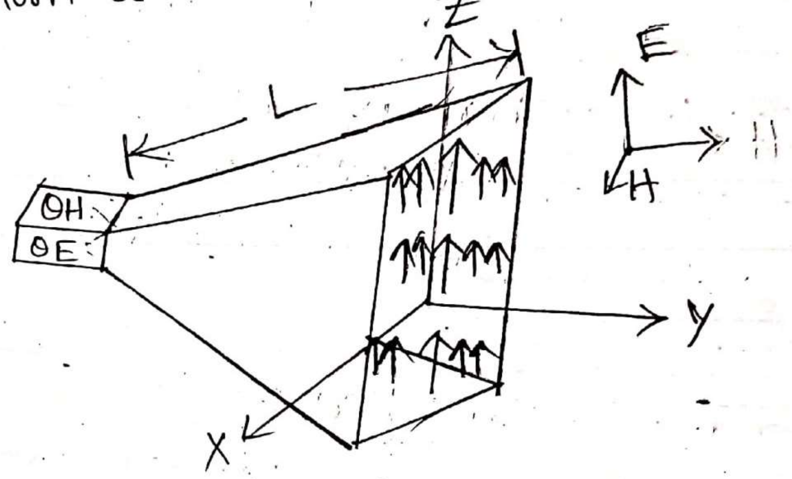
<u>H-plane sectorial Horn</u> :-

* The H-plane sectorial Horn is obtained, when the flaring is done in the direction of magnetic field Vector.



Pyramidal Horn:-

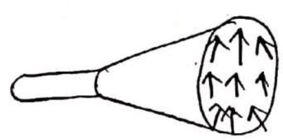
* Pyramidal horn antenna is obtained, when the flaring is done along the both the walls of the rectangular waveguide in the direction of both the electric and magnetic field vectors.
* For pyramidal horn antenna gain is 12-25 dB.



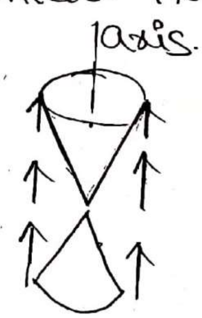
Circular Horn antennas:-

* Circular Horn antennas can be obtained by flaring the walls of circular wave guide.

Conical horn



Biconical Horn



Design characteristics of Horn antennas :-

Let us consider E-plane sectorial Horn. The electromagnetic Horn produces uniform phase front with a larger aperture as compared to waveguide.

* consider an imaginary apex of horn. Assume that there exists a line source which radiates cylindrical waves.
* The constant (or) uniform wave fronts are cylindrical as the waves propagate in the direction radially outwards.

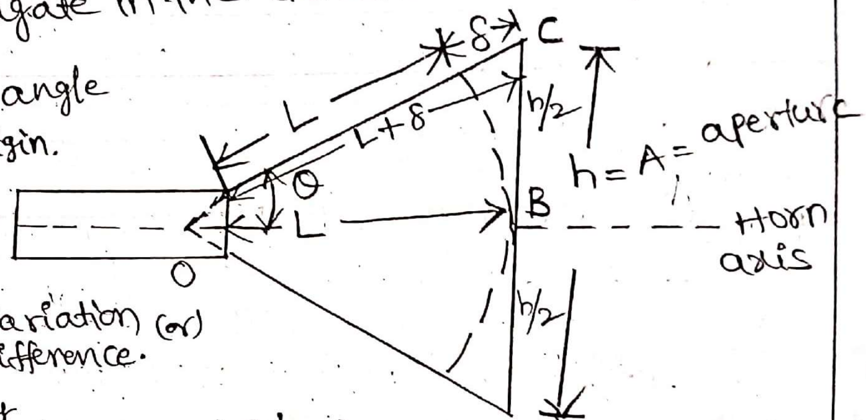
θ = optimum aperture angle

A = aperture , O = origin.

L = axial length

2θ = flare angle.

δ = phase difference variation (or) path difference.



from the geometry,

$$\cos\theta = \frac{L}{L+\delta} \Rightarrow \theta = \cos^{-1}\left(\frac{L}{L+\delta}\right)$$

$$\text{also } \tan\theta = \frac{h/2}{L} = \frac{h}{2L} \Rightarrow \theta = \tan^{-1}\left(\frac{h}{2L}\right)$$

$$\therefore \boxed{\theta = \cos^{-1}\left(\frac{L}{L+\delta}\right) = \tan^{-1}\left(\frac{h}{2L}\right)} \rightarrow (1)$$

from right angle triangle OBC

$$(L+\delta)^2 = L^2 + \left(\frac{h}{2}\right)^2 \quad (\because \text{pythagorous theorem})$$

$$\Rightarrow \cancel{L^2} + \delta^2 + 2L\delta = \cancel{L^2} + \frac{h^2}{4}$$

$$\therefore \delta^2 + 2L\delta = \frac{h^2}{4}$$

if 'δ' is small then δ^2 is Neglected

$$\therefore 2L\delta = \frac{h^2}{4} \quad \text{where } \delta << L$$

$$\boxed{L = \frac{h^2}{8\delta}} \rightarrow (2)$$

Equations ①, ② are called as Design equations.

When flare angle (2θ) is small, the aperture area for a specified Length 'L' becomes small. ∴ the directivity decreases.

* The directivity of maximum value can be obtained at the largest flare angle for which 'δ' does not exceed typical value such as 0.25λ for E-plane horn, 0.32λ for conical horn, 0.40λ for H-plane sectoral horn.

* The directivity of pyramidal and conical horn is highest as compared to other types of horns.

for E-plane horn phase difference up to 72°, for $\delta < 0.2\lambda$

for H-plane horn phase difference upto 135° for $\delta < 0.375\lambda$

* In practical horn antennas flare angle varies from 40° to 15° which gives beam width $= 66^\circ$, Directivity $= 40$, for $L = 6\lambda$;

for $L = 50\lambda$, beam width $= 23^\circ$ and Directivity $= 120$.

for optimum flare horn, the half power beam width is

$$\boxed{\theta_E = \frac{56^\circ\lambda}{a_E} \text{ (or) } \frac{56^\circ\lambda}{h}} \quad \boxed{\theta_H = \frac{67^\circ\lambda}{a_H} \text{ (or) } \frac{67^\circ\lambda}{w}}$$

The relation between directivity and aperture area is

$$\boxed{D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \times \varepsilon_{ap} \times A_p}{\lambda^2}}$$

But $\frac{A_e}{A_p} = \varepsilon_{ap}$ = aperture efficiency.

A_e = effective aperture in m^2

A_p = physical aperture in m^2

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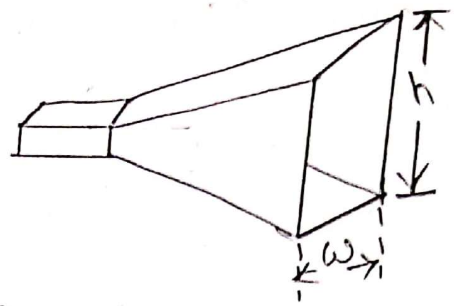
For a rectangular horn

$A_P = a_E \times a_H = h \times w$

Where $a_E = h$ = aperture in E-plane

$a_H = w$ = aperture in H-plane.



$\therefore$ Directivity $D \cong \frac{4\pi \times E_{ap} \times A_P}{\lambda^2}$

$\cong \frac{4\pi \times 0.6 \times A_P}{\lambda^2}$

$\therefore \boxed{D = \frac{7.5 A_P}{\lambda^2}}$

The gain is $\boxed{G_P = \frac{4.5 A_P}{\lambda^2}}$

Features of Horn antennas:-

* Horn antenna is used with waveguide and it is used as radiator.
* It is generally used with paraboloidal antenna as a primary antenna.
* For pyramidal horn, the directivity increases if the flare of the horn is in more than one direction.

Applications of Horn antennas:-

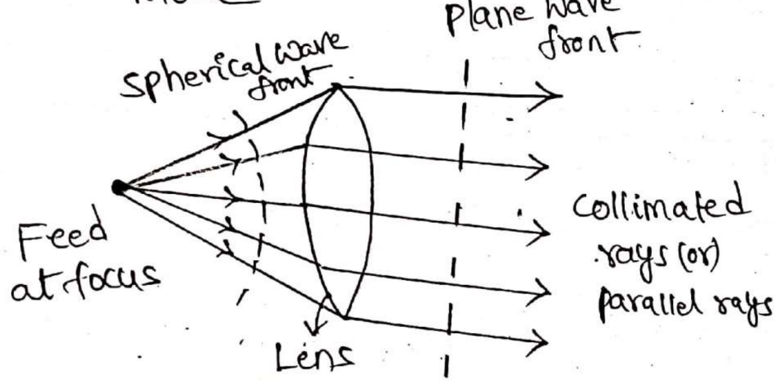
* The horn antenna is used as feed element in antennas such as parabolic reflectors.
* It is the most wide used antenna for measurement of various antenna parameters.

Lens antennas:-

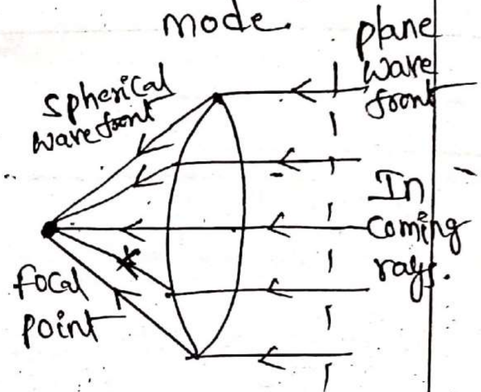
* A lens antenna is an antenna consisting an electromagnetic lens with a feed. It is a three dimensional electromagnetic device having refractive index $n > 1$.

* Its operation is similar to a glass lens used in optics. the lens antenna can be used in Transmitting mode and receiving mode.

Transmitting mode



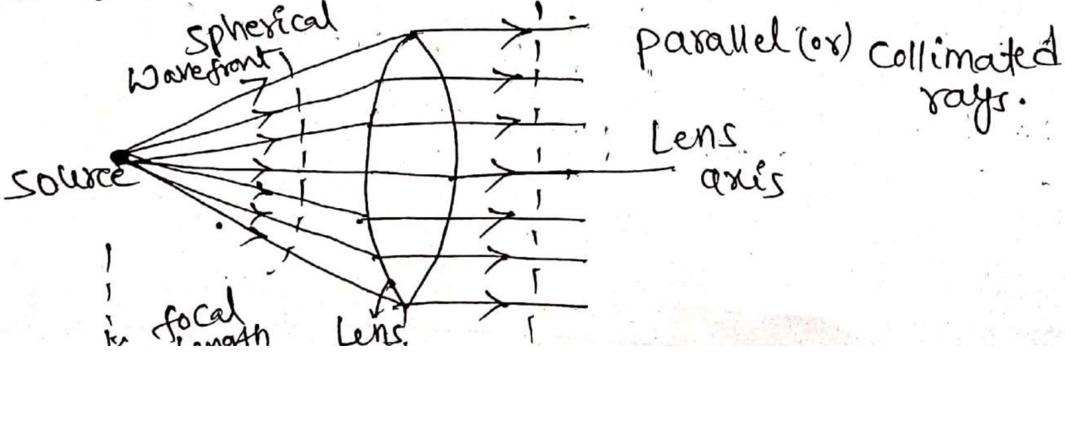
Receiving mode.



* Functions of Lens antennas are
→ It Controls the illumination of aperture
→ It Collimates the electromagnetic rays.
→ It produces directional characteristics.

Principle of Lens antenna:-

→ consider an optical concave lens. If a point source is placed at the focal point of lens, which is along the axis of lens, a focal distance away from lens.



Due to radiation from point source, we get spherical wave front. When the rays travel to the lens refraction takes place, due to the refractive index of lens and rays are collimated, to obtain plane wave front.

* The refraction is more at the edges than refraction at the centre.

* To operate a lens at radio frequencies, a dielectric lens is preferred. Such lens with a point source producing spherical wave front on left hand side of lens to plane wave front on right hand side of lens.

Types of lens antennas:-

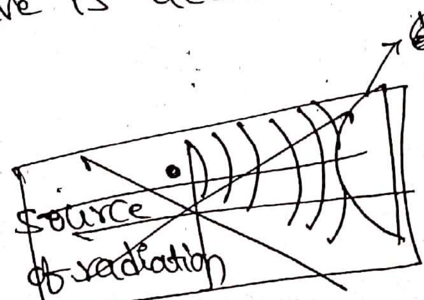
* The main application of lenses is to collimate incident divergent energy and to overcome energy spreading in unwanted directions.

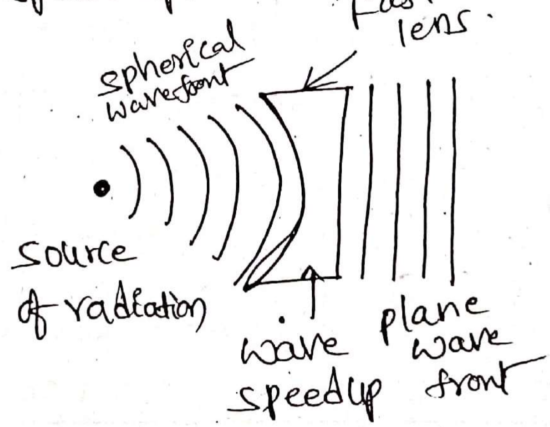
There are 2 types of lens antennas
(i) E-plane metal plate lens (or) Fast lens (or)
(ii) H-plane metal plate lens (or) Delay lens. (or) ~~dielectric~~ dielectric lens.

E-plane metal plate lens :- (Fast lens)

E-plane metal plate lens antenna is the antenna in which

→ The fast lens antenna is decreased by the lens medium electrical path length is decreased (or) speed up. and wave is accelerated (or) speed up.

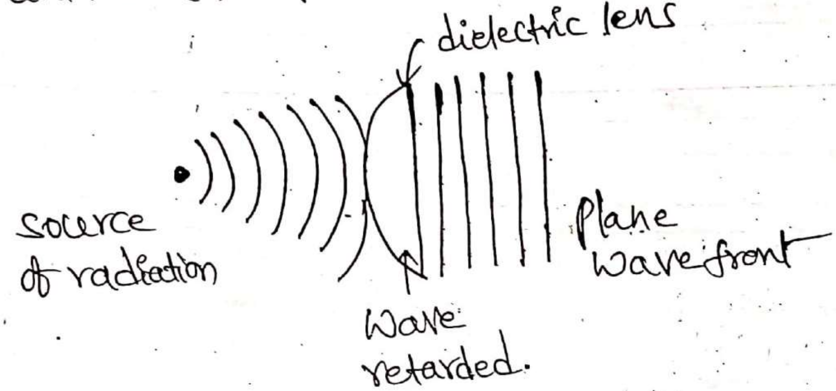




H-plane metal plate lens (or) delay lens
(or)
Dielectric lens antenna.

> The delay lens antenna is the antenna in which the electrical path length is increased by the lens medium and the wave is retarded.

> The delays lens antennas are also called as dielectric lens antenna. (or) H-plane metal plate lens antenna.



Zoning of Lens:-

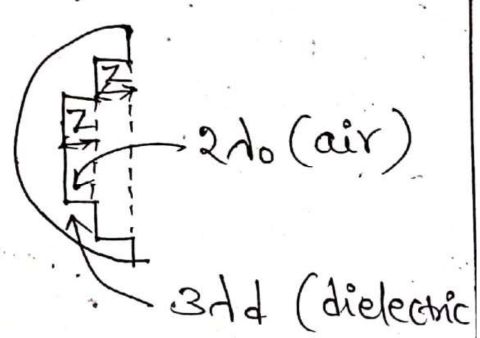
> The weight of the lens can be reduced by removing sections of lens, which is called "zoning" of lens.

Zoning can be classified into two types.

(i) curved surface zoning

(ii) plane surface zoning.

> In general the zoning of lens is carried out in such a way that particular design frequency, the performance of lens antenna is not affected. The zone step is denoted by 'z'.



for dielectric zone step is $3\lambda_d$

for air zone step is $2\lambda_0$.

for λd difference

$$\frac{Z}{\lambda_d} - \frac{Z}{\lambda_o} = 1$$

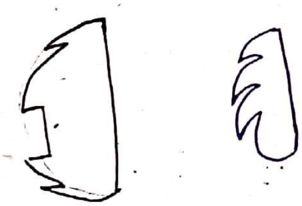
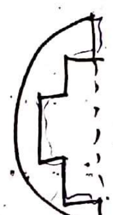
But refractive index $n = \frac{\lambda_o}{\lambda_d} \Rightarrow \lambda_d = \frac{\lambda_o}{n}$

$$\therefore \frac{Z}{(\lambda_o/n)} - \frac{Z}{\lambda_o} = 1$$

$$\Rightarrow \frac{nZ}{\lambda_o} - \frac{Z}{\lambda_o} = 1$$

$$\therefore \frac{(n-1)Z}{\lambda_o} = 1$$

$$\boxed{Z = \frac{\lambda_o}{n-1}}$$

Curved surface Zoning	plane surface Zoning
* As the zoning is done along the curved surface of lens, it is called curved surface zoning 	* As the zoning is done along the plane surface of lens it is called plane surface zoning. 
* It is mechanically stronger than plane surface zoning	* It is mechanically weaker than curved surface zoning.
* It has less weight	* It has more (or) bulky weight.
* The power dissipation of curved surface zoning antenna is less.	* The power dissipation of ~~cur~~ plane surface zoning is more.
*	

Measurement of gain of an antenna:-

> The performance of any antenna can be described interms of figure of merit (or) gain of antenna.

> generally the gain can be measured above 1GHz, free space ranges are used.

> In addition to this microwave Techniques are also used for gain measurements.

> Antenna gains are not measured below 1MHz.

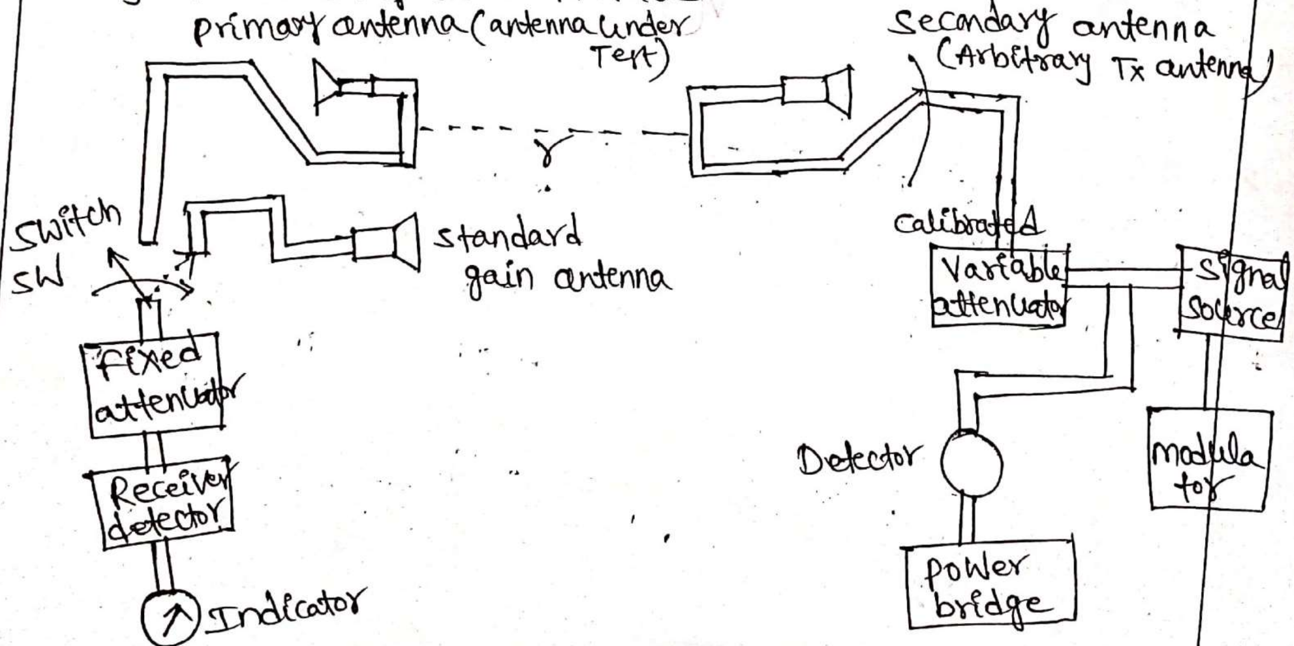
Basically there are two standard methods for measurement of gain.

(i) Gain- comparison (or) Direct comparison method.
(ii) Absolute gain method.

$$\text{Gain} = \frac{\text{Max radiation Intensity (Test or subject antenna)}}{\text{Max radiation Intensity (Reference antenna)}}$$

Direct Comparison Method:-

> At high frequencies the gain measurement is done using direct comparison method.



In this Method the gain measurement is done by comparing the strengths of the signals Transmitted or Received by the antenna under Test and standard gain antenna.

→ the antenna whose gain is accurately known that is called as "standard gain" antenna. generally standard gain antenna is Horn antenna.

→ This Method uses two antennas termed as primary antenna and secondary antenna.

→ the primary antenna consists of two different antennas seperated through a switch SW. The first primary antenna is standard gain antenna and second primary is subject antenna under Test

→ These two primary antennas are located at sufficient distance of seperation.

The two steps for gain comparison method are

* through the switch SW, the first standard gain antenna is connected to Receiver. the antenna is adjusted in the direction of secondary antenna to have maximum signal Intensity. The i/p connected to the secondary or Transmitting antenna is adjusted to require level. for this i/p corresponding primary antenna reading is recorded at Receiver. corresponding attenuator and power bridge readings are recorded as A_1 and P_1.

* secondly the antenna under test is connected to Receiver by changing the position of switch SW. To get the same reading at Reciver, the attenuator is adjusted. Then corresponding Attenuator and power bridge readings are A_2 and P_2.

Case I :- If $P_1 = P_2$, then no correction need to applied and the gain of the subject antenna under is given by

power gain $= G_P = \frac{P_2}{P_1}$, where P_1 and P_2 are power levels.

Taking logarithms on b/s. we get

$$\log_{10} G_P = \log_{10}\left(\frac{P_2}{P_1}\right) = \log_{10} P_2 - \log_{10} P_1$$

(ie) $\boxed{G_P(dB) = P_2(dB) - P_1(dB)}$

Case II :- If $P_1 \neq P_2$ then the correction need to be included.

Let $\frac{P_1}{P_2} = P$ then

$$\log_{10}\left(\frac{P_1}{P_2}\right) = P(dB)$$

Power gain is given by

$$G = G_P \times \frac{P_1}{P_2}$$

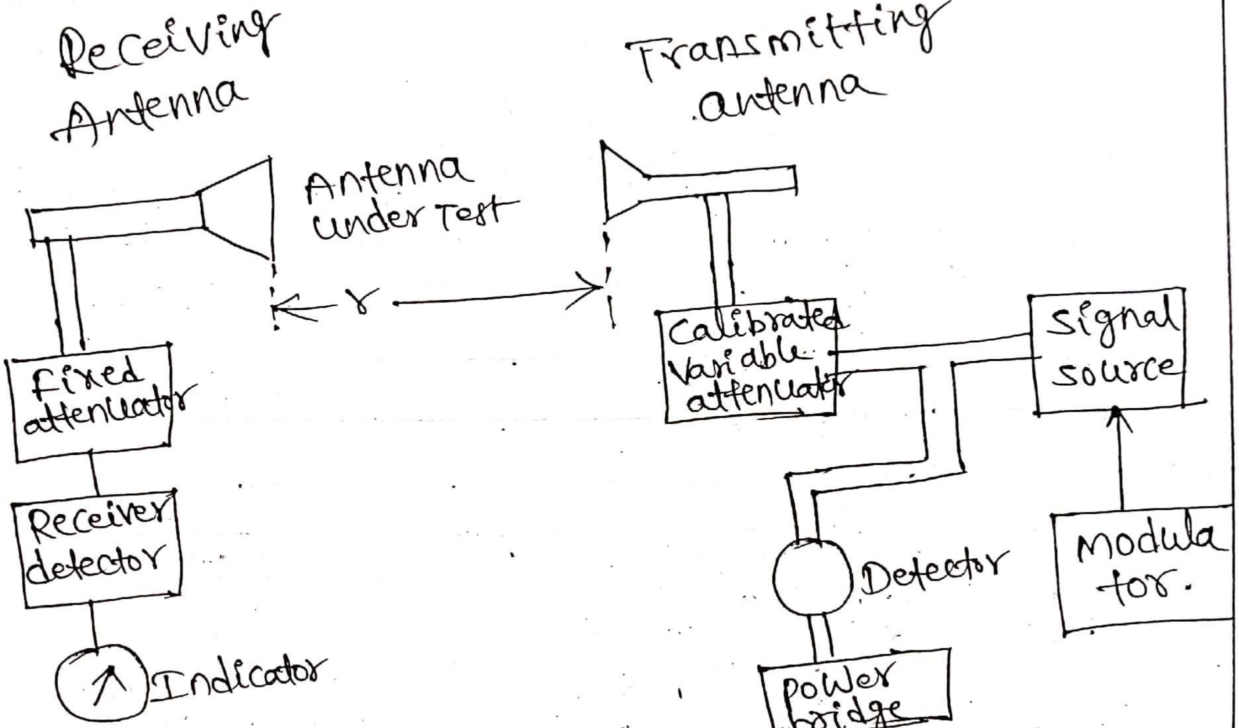
Taking log on both sides

$$\log_{10} G = \log_{10}\left(G_P \cdot \frac{P_1}{P_2}\right) = \log_{10} G_P + \log_{10}\frac{P_1}{P_2}$$

$$\therefore \boxed{G(dB) = G_P(dB) + P(dB)}$$

Measurement of Absolute gain method

Consider two identical antennas seperated by distance 'r'



Let the transmitted power be denoted by 'Pt' and Received power be denoted by Pr. effective aperture are of Aet for Transmitting antenna, Aer for Receiving antenna.

$$A_{et} = A_{er} = \frac{G_D \lambda^2}{4\pi} \qquad (\because G_D = \text{Directive gain (or) = directivity})$$

from friis Transmission equation

$$\frac{P_r}{P_t} = \frac{A_{er} \cdot A_{et}}{\lambda^2 \cdot r^2} = \left(\frac{G_D \lambda^2}{4\pi}\right)\left(\frac{G_D \lambda^2}{4\pi}\right)\frac{1}{\lambda^2 r^2}$$

$$\frac{P_r}{P_t} = \left(\frac{G_D \lambda}{4\pi r}\right)^2 \Rightarrow \frac{G_D \lambda}{4\pi r} = \sqrt{\frac{P_r}{P_t}}$$

$$\therefore \boxed{G_D = \frac{4\pi r}{\lambda}\sqrt{\frac{P_r}{P_t}}}$$

Measurement of directivity :- (3-Antenna method)

Directivity is defined by

$$D = \frac{\text{Max Radiation Intensity}}{\text{Avg Radiation Intensity}}$$

$$\boxed{D = \frac{U_{max}}{U_{avg}}} \quad \text{(OR)}$$

$$D = \frac{r^2 P_d(max)}{\left(\frac{P_{rad}}{4\pi}\right)} \qquad \left(\because U_{max} = r^2 P_d(max),\ U_{avg} = \frac{P_{rad}}{4\pi}\right)$$

$$\Rightarrow D = \frac{P_d(max)}{\left(\frac{P_{rad}}{4\pi r^2}\right)} \qquad \left(\because P_{avg} = \frac{P_{rad}}{4\pi r^2}\right)$$

$$\therefore \boxed{D = \frac{P_d(max)}{P_{avg}}}$$

$$D = G_{D\,max} = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi |E_{max}|^2}{\int_0^{2\pi}\int_0^{\pi} |E(\theta,\phi)|^2 \sin\theta \, d\theta \, d\phi}$$

$$= \frac{4\pi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{|E(\theta,\phi)|^2}{|E_{max}|^2} \sin\theta \, d\theta \, d\phi}$$

$$\therefore \boxed{D = G_{D\,max} = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} f_n(\theta,\phi) \sin\theta \, d\theta \, d\phi}}$$

where $f_n(\theta,\phi)$ = normalized field Radiation.

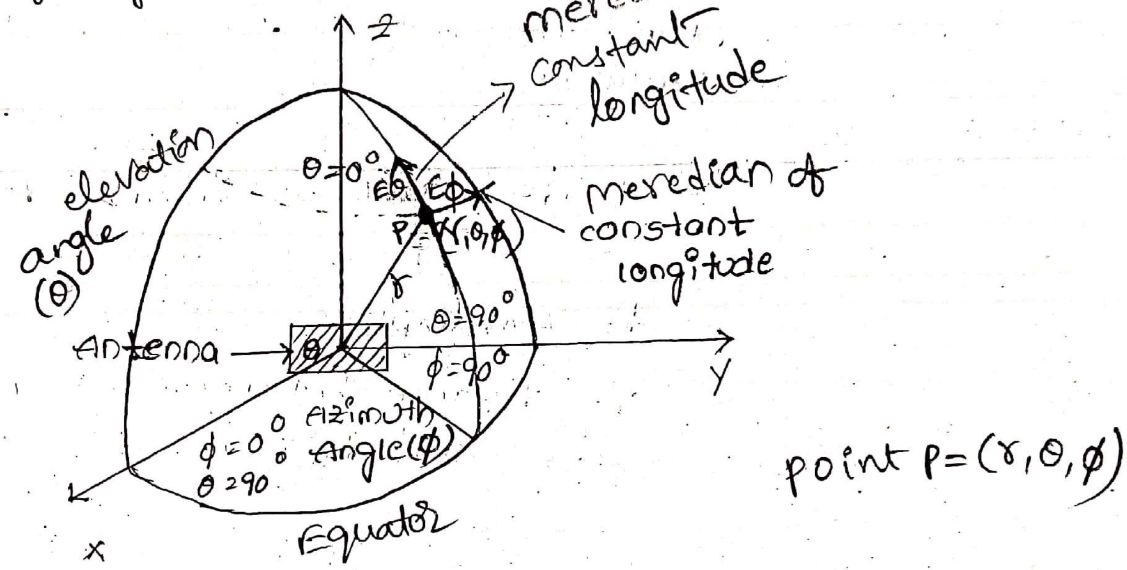
and $$\boxed{D = \frac{41{,}253}{\theta_E \times \theta_H}}$$

Set up:-

Radiation pattern Measurement:-

→ Radiation pattern of a Transmitting antenna is described as the field strength or power density at a fixed distance from the antenna as function of direction

→ The Test antenna is assumed to be placed at the origin of spherical coordinates.



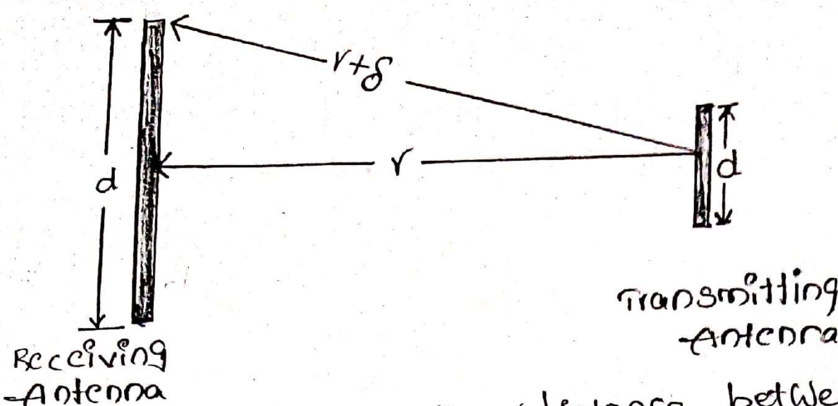
→ For most antennas it is generally necessary to take radiation pattern in XY plane (Horizontal plane) and XZ plane (vertical plane).

Distance criteria:-

→ In order to obtain accurate far field, the distance between primary and secondary antenna must be large.

→ If the distance between two antennas is very much small, then near field pattern is obtained.

→ The phase difference between centre and edges of Receiving antenna shown in the figure.



→ under this condition, the distance between primary and secondary antenna should be

$$\boxed{r \geq \frac{2d^2}{\lambda}}$$

where d = maximum linear dimension of either (Both) antennas.

λ = Wavelength

r = distance between Transmitter and Receiver

→ ALL THE BEST ←

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ECE

UNIT-6 (AWP)

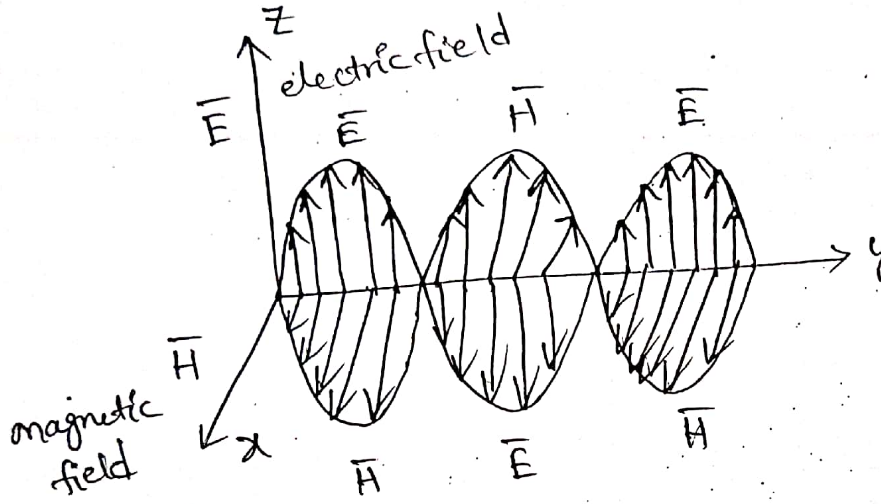
WAVE PROPAGATION :-

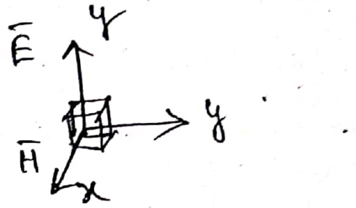
S.M ①

Wave propagation :- The electromagnetic waves (or) Radio waves propagating from Transmitting antenna to Receiving antenna.

→ The power Radiated by the current carrying conductor then propagates in the free space in the form of EM waves. These Electromagnetic waves are oscillating in nature. In the free space, EM waves travel at the speed of light.

→ The speed of light is $c = 3\times10^8$ m/sec (or) $c = 3\times10^{10}$ cm/sec





Frequency Ranges :-

Symbol	frequency Range	Wave length (meters)	Type of Propagation
ELF	< 300 Hz	> 1000 Km	Earth - Ionosphere Wave guide propagation
VLF	300 Hz - 3 KHz	1000 Km - 100 km	Ground wave propagation
ULF	3K - 30 KHz	100 km - 10 km	
LF	30K - 300 KHz	10 km - 1 km	
MF	300 KHz - 3 MHz	1 km - 100 m	Skywave propagation
HF	3 MHz - 30 MHz	100 m - 10 m	
VHF	30 MHz - 300 MHz	10 m - 1 m	Space wave propagation / Tropospheric scattering / LOS propagation.
UHF	300 MHz - 3 GHz	1 m - 100 mm	
SHF	3 GHz - 30 GHz	100 mm - 10 mm	
EHF	30 GHz - 300 GHz	10 mm - 1 mm.	

Radar frequency Band According to IEEE Standards :-

Letter Designation	Frequency Band (GHz)
L	1-2 GHz
S	2-4 GHz
C	4-8 GHz
X	8-12 GHz
Ku	12-18 GHz
K	18-27 GHz
Ka	27-40 GHz
V	40-75 GHz
W	75-110 GHz
mm	110-300 GHz

Structure of atmosphere :-

→ In the Radio wave propagation, the earth's environment between the Transmitting and Receiving antennas play a very Important Role.

→ The atmosphere of the earth mainly consists of 3 Regions.

(i) Troposphere.

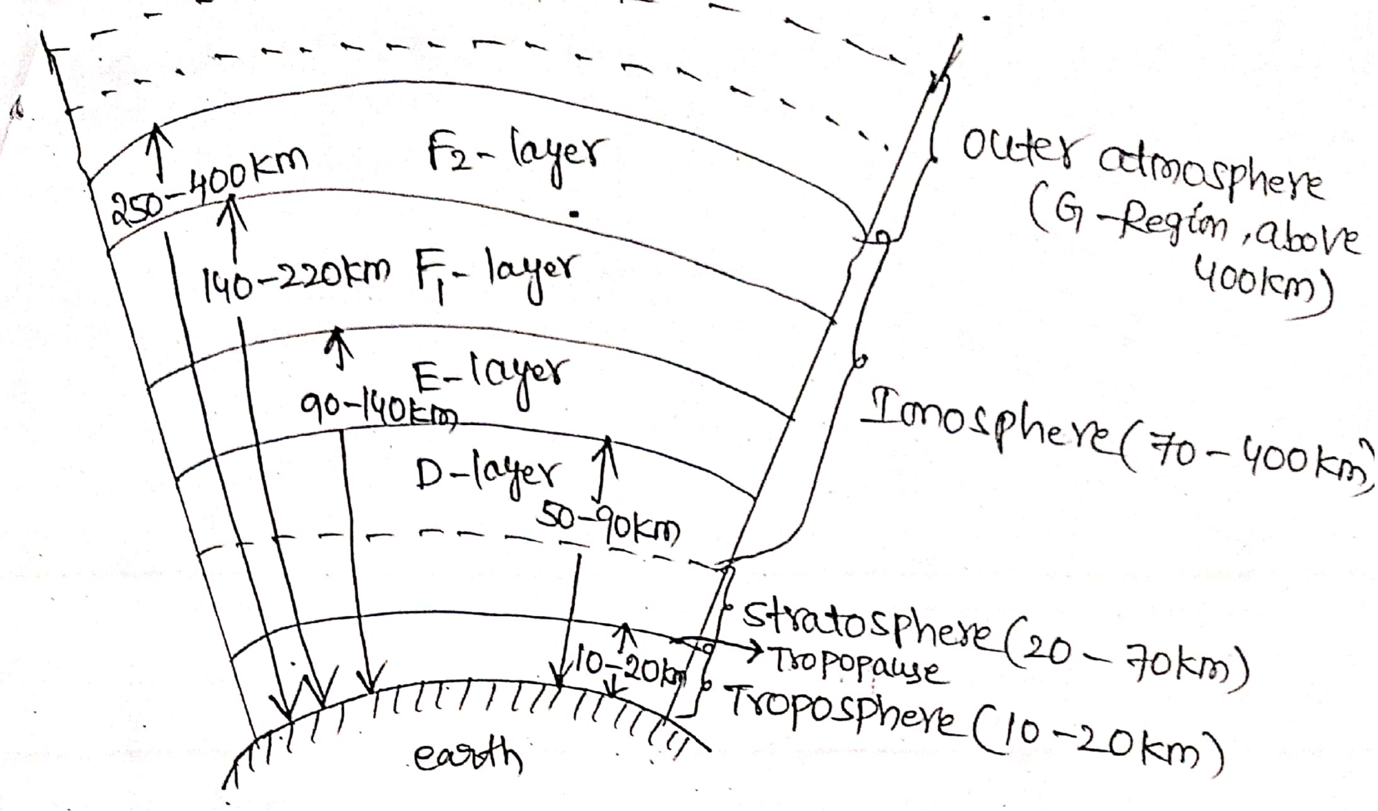
(ii) stratosphere

(iii) Ionosphere.

→ The Troposphere is the nearest Region of the atmosphere to the earth's surface at about 10 to 20 km above the earth surface.

→ The stratosphere is the Region which is in between 20 km to 70 km of height from the earth's surface.

→ The Ionosphere is the last Region, which extends approximately 70km to 400km above the earth's surface.



Structure of Troposphere :-

→ This is the nearest Region in the atmosphere from the earth's surface around the 10km to 20km above the earth's surface.

The troposphere is also called "Region of change".

→ At a certain height called the critical height above troposphere the temperature remains constant for narrow region and then Increases afterwards.

→ The region between the top of troposphere and the beginning of stratosphere is called "Tropopause".

→ The Region between 20km to 70km above the earth's surface is called "Region of calm" (or) "stratosphere".

Structure of Ionosphere :-

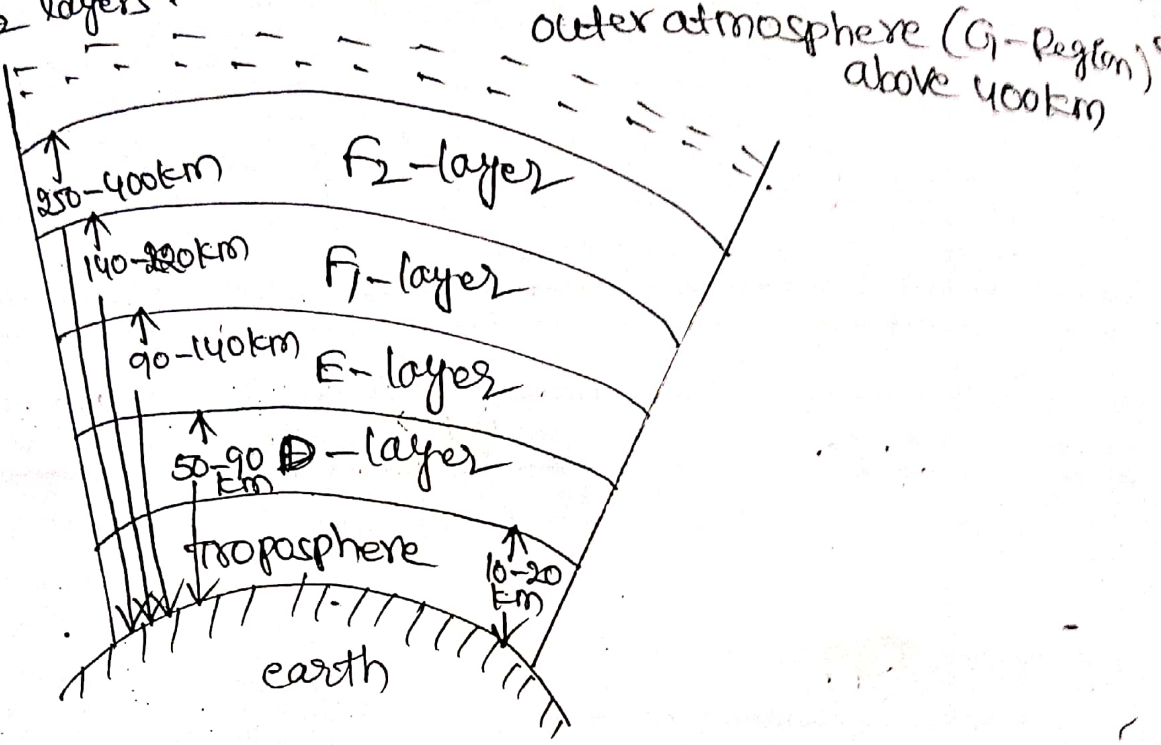
→ Ionosphere is the upper portion of the atmosphere of the earth.

→ It gets heated due to the large absorption of large energy radiated by the sun. After heating it get Ionized.

→ This Region is located about 70km above the earth surface and upto 400km.

→ There are different Variations in properties of the atmosphere such as temperature, pressure, density, composition etc.

→ Due to the Variation in these properties and the absorption of different Radiations by the Ionosphere, It becomes irregular distribution and thus four main layers namely D layer, E layer, F1 and F2 layers.



D-layer:- The D-layer is located about 50 to 90 km above the surface of earth and it is nearest layer to the earth's surface.

→ Its thickness is about 10km.

→ This layer is Ionized by photo Ionization of O_2 molecules.

→ The Ionic density about 400/cm^2 and electron density of maximum value.

→ This layer reflect Very low frequency (VLF) and Low Frequency (LF) waves.

→ The critical frequency is about 100KHz. D-layer present at Day time only.

E-layer:- The E-layer is located about 90-140 KM above the earth surface.

→ Thickness is about 25 km.

→ This layer is Ionized by all gases by X-ray radiation takes place.

→ During night time its Ionization is Weak.

→ The maximum electron density is about $4\times10^5/cm^3$ and is at height of 100 km.

→ It is useful for high frequency (HF) waves during day time.

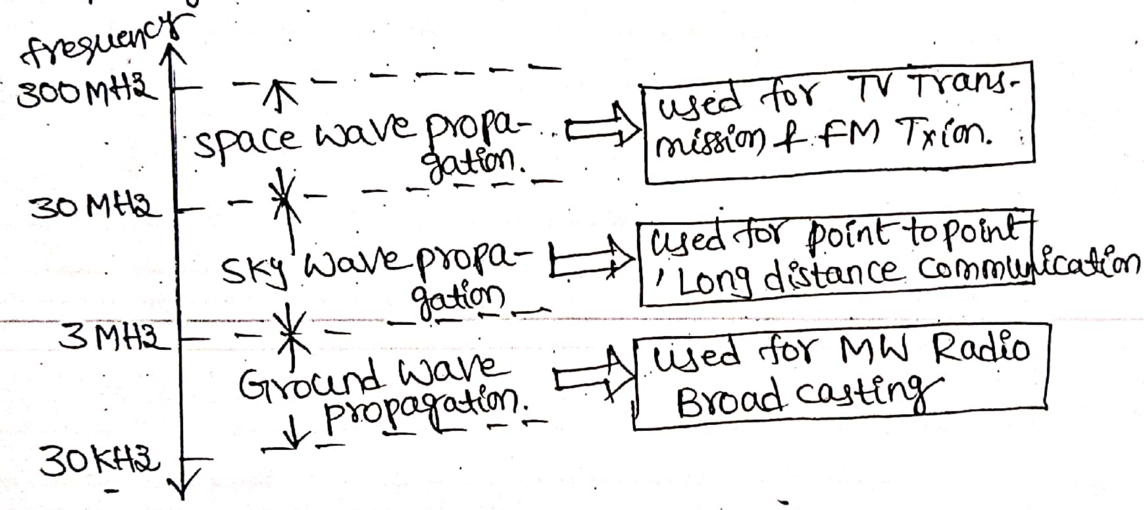
→ Critical frequency is about 3 to 5 MHz. It provides better Reception during night time.

F layer:- The F-layer is located at the height of 140 to 400km and it is mainly combination of F_1 layer (140 - 220km) and F_2 layer (250 to 400km). During night F_1-layer combines with F_2-layer and at height of 140-300 km, we get F layer.

→ This layer only ionized during day time as well as night time.
→ The maximum election density is 220km approximately
→ Critical frequency is 5 to 12 MHz.
→ The F_1 layer reflects the high frequency waves.
→ The F_2 layer reflects the high frequency Radio waves.

Modes of propagation:-

There are 3 different modes of propagation.
1) Ground wave propagation 2) Sky wave propagation 3) Space-wave propagation.



Ground wave propagation:- [surface wave]

→ The waves which are propagated near the earth's surface are called "ground waves".
→ The frequency range of ground wave propagation is 300KHz to 3MHz.
→ The ground wave propagation is possible when the Transmitting and Receiving antenna both are closed to the earth's surface
→ This type of propagation is used for MW, Radio Broad casting.
→ The ground waves are vertically polarized waves, It should require high power for Transmission.

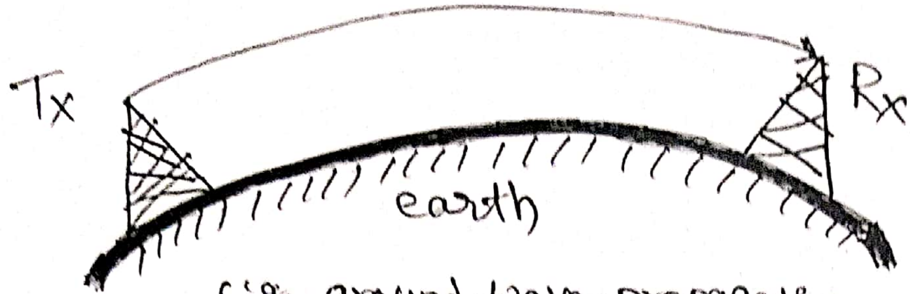


fig:- ground wave propagation.

→ The ground wave propagation is about LF and MF frequencies.

→ The ground wave is a vertically polarized wave that travels along the surface of the earth. For the ground wave propagation, vertical antennas are useful. If a horizontally polarized wave is propagated as ground wave, then the electric field of a wave gets short circuited due to conductivity of the earth. Hence the ground wave is always a vertically polarized wave. Hence, as the ground wave travels away from the transmitting antenna, it gets attenuated.

$$\boxed{E = \frac{120\pi\, h_t \cdot h_r \cdot I_s}{\lambda d} \; v/m.}$$

Where $120\pi = 377\,\Omega$ = Intrinsic impedance of free space.

h_t and h_r = Effective heights of the transmitting and receiving antennas respectively.

I_s = Antenna current

λ = wave length.

d = Distance at a point from the transmitter.

Wave tilt :- Wave tilt is defined as the angle Normal to the ground wave and electric plane wave. Where the ground wave is vertically polarized.

Sky wave propagation :- (Iono-spheric propagation)

The skywaves are of practical importance for every long radio communications at medium and high frequencies. The sky wave propagation is about the frequency range of 3-30 MHz.

→ Sky wave propagation is also called as Ionospheric Propagation.

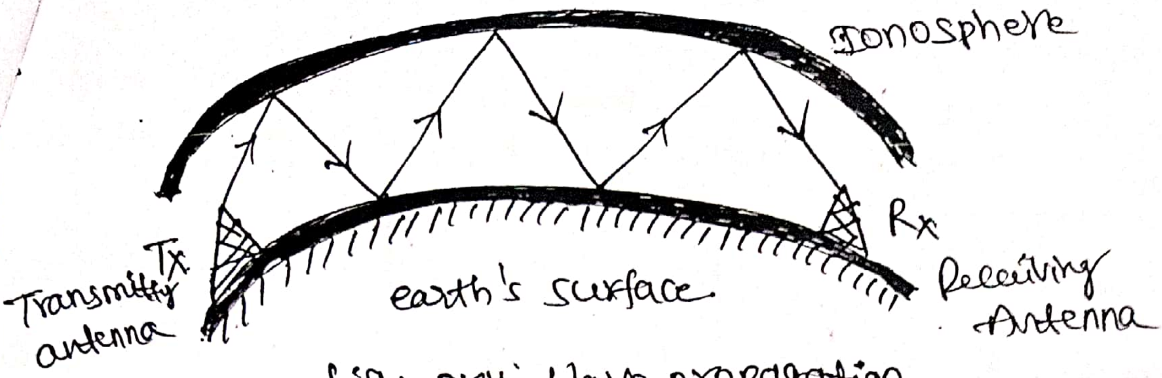


fig:- Sky-Wave propagation

→ this mode is used in HF band international broad casting.

→ In this mode the EM waves transmitted by the transmitting antenna reach the receiving antenna.

→ At very long distance away from transmitting antenna, after the reflection from the ionized region. in the upper part of the atmosphere of the earth.

→ This part is called ionosphere and it is located above earth's surface at about 70km to 400km height.

→ Using the sky wave propagation, a long distance point to point communication is possible and hence it is also called point to point propagation (or) point to point communication.

→ The sky wave propagation is also called as iono spheric propagation. Because the waves reach the receiver after reflecting from earth to Iono sphere.

→ EM waves directed in upword at some angle from earth surface are called as sky waves.

→ Sky wave propagation is used for long distance communication.

→ Iono sphere is the upper portion of atmosphere between 50km to 350km about the earth.

Maximum usable frequency (MUF)

→ It is defined as the sky waves are sent by the maximum frequencies at some incidence angles towards the Ionosphere then these waves will again reflected back to the earth by Ionospheric Layers. Maximum usable frequency exists in sky wave propagation. $\boxed{f_{MUF} = \frac{f_{cr}}{\cos\phi_i}}$ (or) $\boxed{f_{MUF} = \sec\phi_i \, f_{cr}}$

critical frequency :- [fcr]

critical frequency is defined as the highest frequency that, finally effe... be reflected back to the earth by a particular layer for a vertical Incidence. It is denoted by 'f_{cr}'

→ The critical frequency is different for different layers.

$$\boxed{f_{cr} = \sqrt{81 N_{max}} = 9\sqrt{N_{max}}}$$

where N_{max} is the no. of electrons expressed per cubic meter and the critical frequency f_{cr} is in Mega Hertz.

Mechanism of Reflection and Refraction :-

→ Basically the Reflection and Refraction of the Radio waves is the function of the frequency of the wave.

→ for very low frequencies the wavelengths are Larger and for very high frequencies the wavelengths are very small.

(i) Reflection at Low Frequencies :-

The wavelength for low frequencies is very large. Thus the changes in the Ionization density are considerably large. The layer of Ionosphere acts as a dielectric having reflection coefficient given by

$$R_1 = \frac{\cos\theta - \sqrt{\left(\epsilon_r' + \frac{\sigma}{j\omega\epsilon_0}\right) - \sin^2\theta}}{\cos\theta + \sqrt{\left(\epsilon' + \frac{\sigma}{j\omega\epsilon_0}\right) - \sin^2\theta}}$$

$$R_V = \frac{\left(\epsilon_r' + \frac{\sigma}{j\omega\epsilon_0}\right)\cos\theta - \sqrt{\left(\epsilon_r' + \frac{\sigma}{j\omega\epsilon_0}\right) - \sin^2\theta}}{\left(\epsilon_r' + \frac{\sigma}{j\omega\epsilon_0}\right)\cos\theta + \sqrt{\left(\epsilon_r' + \frac{\sigma}{j\omega\epsilon_0}\right) - \sin^2\theta}}$$

Where $\epsilon_r' = 1 - \frac{Ne^2}{m\epsilon_0(\omega_0^2 + \omega^2)}$ and $\sigma = \frac{Ne^2\omega_0}{m(\omega_0^2 + \omega^2)}$

Where N = electron density $/m^3$

e = electron charge $= 1.6 \times 10^{-19}$ C, m = mass of electron $= 9 \times 10^{-31}$ kg

ω_0 = frequency of angular $= 2\pi f_0$

that Finally we concludes that, at lower frequencies the reflection coefficient depends upon the
* Frequency of wave
* Angle of Incidence of wave
* polarization of wave (horizontal or vertical) (5)

ii) Refraction at High frequencies:-

→ At high frequencies the wavelength is very small. The analysis at high frequencies carried out using ray optics if the change in the phase velocity is within short wavelength is very small.

the phase velocity of the wave within a medium is given by

$$V_p = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \times \frac{1}{\sqrt{\mu_r \epsilon_r}} \longrightarrow (1)$$

$$\Rightarrow \boxed{V_p = \frac{c}{\sqrt{\mu_r \epsilon_r}}}$$ where $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ = velocity of light in freespace

Assume that the permeability of the Ionosphere is unchanged due to the presence of electrons, hence $\boxed{\mu_r = 1}$.

$$\therefore \boxed{V_p = \frac{c}{\sqrt{\epsilon_r}}} \longrightarrow (2)$$

From equation (2) It is clear that the phase velocity depends on ϵ_r.

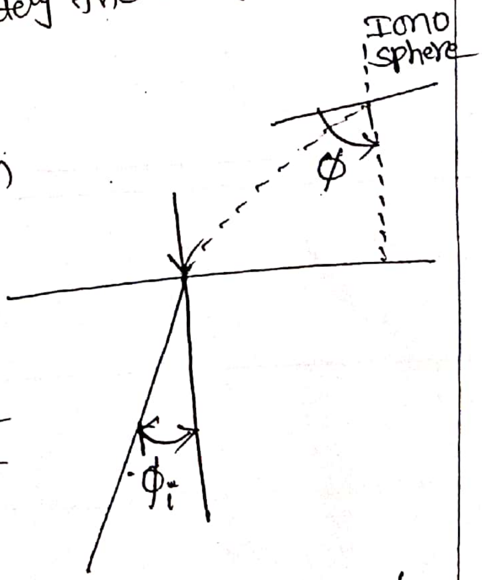
→ The phase velocity also depends on electron density N.

→ Hence for the high frequency, the wavelength is shorter so that the change in electron density is small and ultimately the changes in phase velocity are smaller.

→ Now consider the wave is Incident on the lower edge of Ionosphere without any Reflection

→ But as the wave penetrates the Ionosphere, the wave follows the curved path and it moves away from region of greater electron density.

→ Thus at any point on the curved path the angle between the path and the normal at that point can be obtained by using the snell's law.



Application of Snell's law

According to snell's law

$$\sin\phi_i = n \sin\phi \quad (or) \quad n = \frac{\sin\phi_i}{\sin\phi}.$$ (n = refractive Index.)

$$\Rightarrow \sin\phi = \frac{\sin\phi_i}{n}$$

→ The refractive Index of medium is given by

$$n = \frac{\text{Velocity of Light in free space}}{\text{phase Velocity in the medium}} = \frac{c}{V_p}$$

$$\therefore n = \frac{c}{\frac{c}{\sqrt{\epsilon_r}}} = \sqrt{\epsilon_r} \qquad \left(\because \text{from eq } 2 \quad V_p = \frac{c}{\sqrt{\epsilon_r}}\right)$$

$$\boxed{n = \sqrt{\epsilon_r}}$$

Where

$$\epsilon_r = \left(1 - \frac{Ne^2}{\epsilon_0 m \omega^2}\right) \rightarrow 4$$

for electron $m = 9 \times 10^{-31}$ kg = mass of electron

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m

e = charge of electron = 1.6×10^{-19} C

$$\epsilon_r = \left(1 - \frac{N(1.6 \times 10^{-19})^2}{8.854 \times 10^{-12} \times 9 \times 10^{-31} \times (2\pi f)^2}\right)$$

$$\epsilon_r = 1 - \frac{81N}{f^2} \qquad \text{where } \omega = 2\pi f$$

$\therefore$ The Refractive Index $\boxed{n = \sqrt{\epsilon_r} = \sqrt{1 - \frac{81N}{f^2}}} \rightarrow 5$

since, if $\phi_i = 0$, then $\sin\phi_i = n \sin\phi$.

$\sin 0 = n \sin\phi$

$n = 0$.

$$\therefore \sqrt{1 - \frac{81N}{f^2}} = 0 \Rightarrow 1 - \frac{81 N_{max}}{f_{cr}^2} = 0$$

$$\Rightarrow \frac{81 N_{max}}{f_{cr}^2} = 1$$

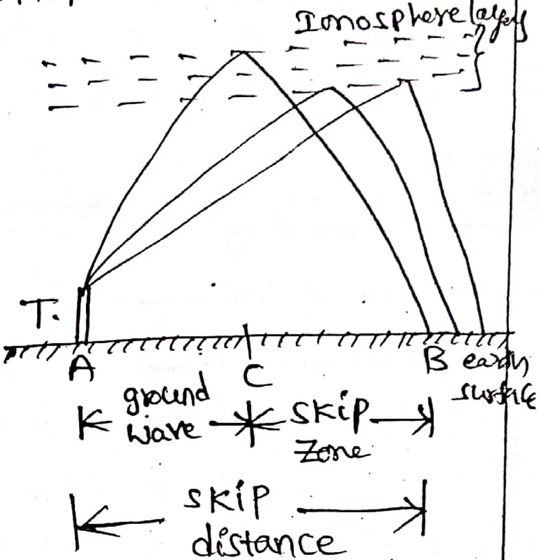
$$81 N_{max} = f_{cr}^2$$

At $\phi_i = 0$, the critical frequency exists. $\boxed{f_{cr} = \sqrt{81 N_{max}}}$ → critical frequency

Skip distance :- (d_{skip})

The skip distance is the shortest distance from the Transmitter, measured along surface of the earth, at which a sky-wave of fixed frequency will return back from Ionosphere to earth.

For a given frequency $f = f_{MUF}$, the skip distance can be calculated as follows.



$$f_{MUF} = f_{cr}\sqrt{1+\left(\frac{D_{skip}}{2h}\right)^2}$$

$$\Rightarrow \frac{f_{MUF}}{f_{cr}} = \sqrt{1+\left(\frac{D_{skip}}{2h}\right)^2}$$

$$\Rightarrow \left(\frac{f_{MUF}}{f_{cr}}\right)^2 = 1+\left(\frac{D_{skip}}{2h}\right)^2$$

$$\Rightarrow \left(\frac{f_{MUF}}{f_{cr}}\right)^2 - 1 = \left(\frac{D_{skip}}{2h}\right)^2$$

$$\therefore \frac{D_{skip}}{2h} = \sqrt{\left(\frac{f_{MUF}}{f_{cr}}\right)^2 - 1}$$

$$\boxed{D_{skip} = 2h\sqrt{\left(\frac{f_{MUF}}{f_{cr}}\right)^2 - 1}}$$

Fading :- Fading is defined as the fluctuation in the received signal strength at the Receiver (or) a random variation in the received signal.

Fading may be classified interms of duration of variation in signal strength as
→ Rapid fluctuations
→ short term fluctuations
→ Long term fluctuations.

The various types of fading are as follows.

1. Selective fading
2. Absorption fading
3. Interference fading
4. polarization fading
5. Skip fading.

→ The fading is caused due to interference between two wav
of different path lengths.

→ Fading is caused due to variations in height and density of the Ionizing in different layers.

1. Selective fading:- It is more dominant at high frequencies for which sky wave propagation is used. The selective fading produces serious distortion of modulated signal. The fading frequency selective, hence the portion or frequency also be faded Independent

2. Absorption fading :- This type of fading occurs due to the variations of single strength with the different amount of absorption of waves absorbed by the Transmitting medium.

3. Interference fading :- It is the fading produced because of upper and lower rays of the sky wave interfering with each other. This is the most serious fading.

4. Polarization fading :- When the sky wave reaches after the reflection, the state of polarization is changing. The polarization of sky wave coming down changes because of the superposition ~~and~~ of the ordinary and extra ordinary waves, which are oppositely polarized. Thus polarization of wave changes.

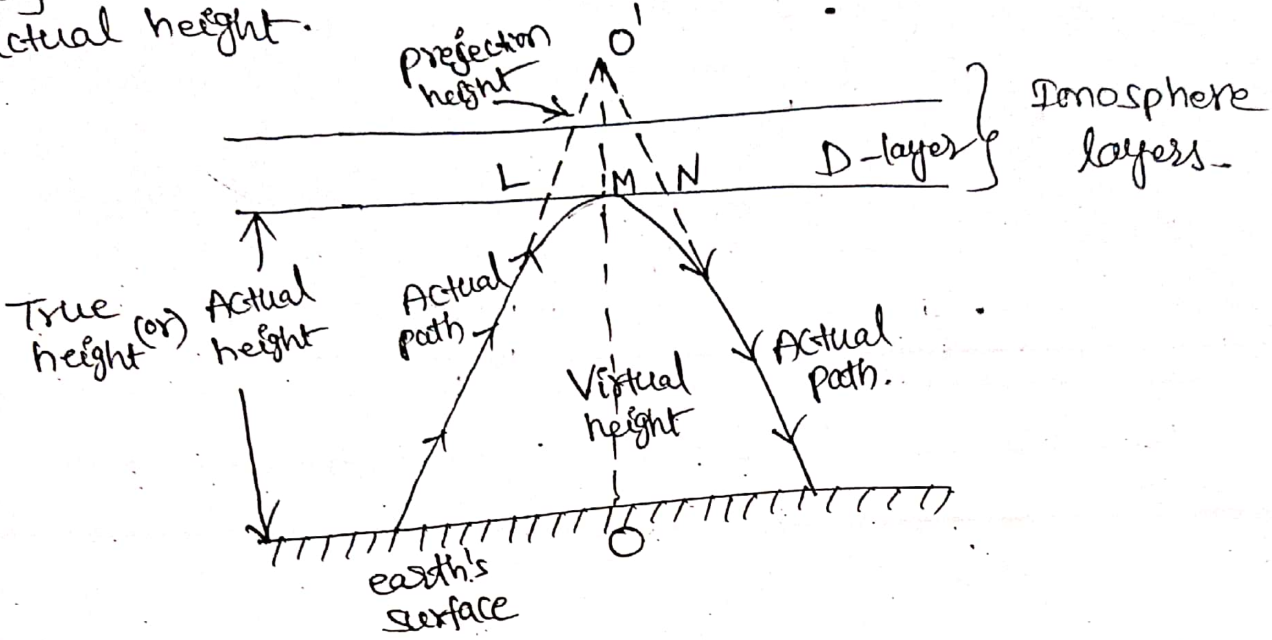
5. Skip fading:- At distances near the skip zone, the fading occurs, which is called skip fading.

→ To minimize the skip fading, the most common method is to use automatic voltage control and Automatic Gain control (AVC or AGC)

Actual height, Virtual height:-

Actual height:- The height at which the wave bending down to the earth surface. It is called as Actual height (or) True height

Virtual height :- It is defined as the height to which, a short pulse of energy Transmitt along vertically upwards and a wave travelling with the speed of light. The Virtual height is greater than actual height.

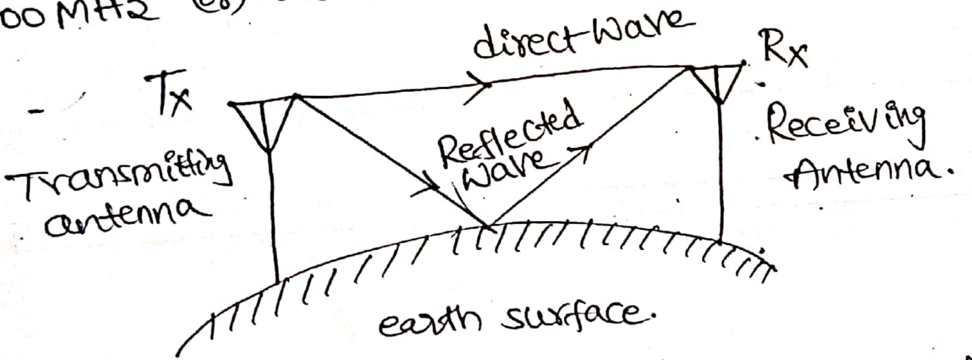


Space wave propagation :-

→ The radio waves which are having high frequencies are called as space waves.

→ space waves are the combination of direct wave and Reflected wave.

→ The frequency range of space wave propagation is about 30MHz to 300 MHz (or) above 30MHz frequencies.



→ The space wave propagation is composed of direct wave propagation and reflected wave propagation.

→ The space wave propagation is through troposphere, hence such propagation is limited to few hundreds of kilometers.

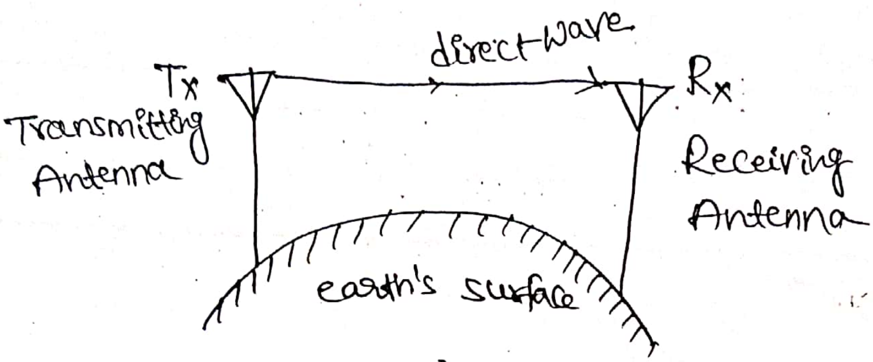
→ The space wave propagation propagates through the frequency Bands of HF and VHF frequency Bands.

LOS propagation :- [Radio Horizon] :-
↳ line of sight.

→ The LOS propagation is also called as "Direct Wave propagation"

→ LOS propagation is a characteristic of electromagnetic radiation (or) Accoustic Wave Radiation.

→ The frequency Range of LOS propagation is above 30 MHz



→ The Transmitter and Receiver are placed within the line of sight distance.

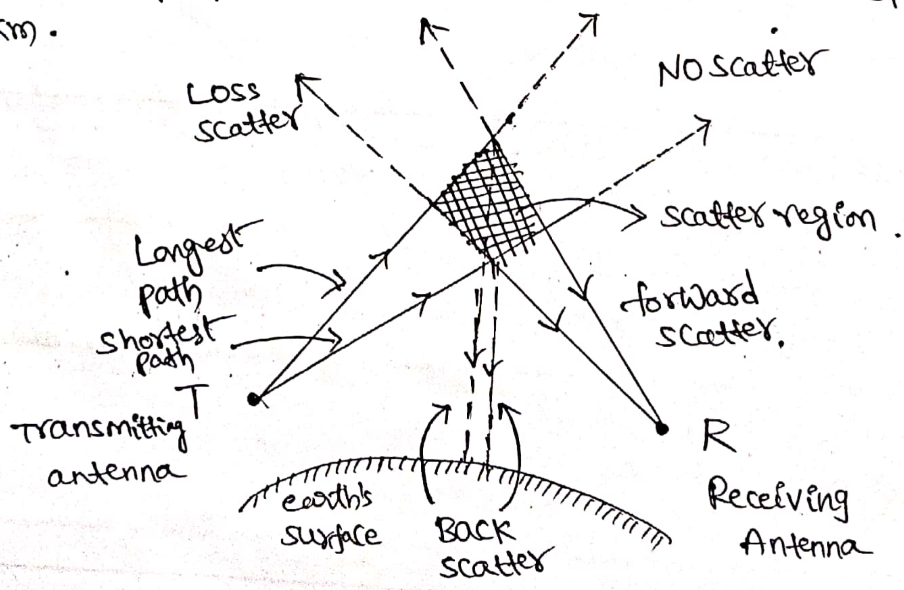
→ The waves are travelling in a direct path from transmitter to Receiver.

→ The Refraction takesplace in the LOS propagation.

Tropospheric scattering propagation :- (forward scattering propagation).

→ The tropospheric propagation (or) tropospheric scattering propagation is nothing but the propagation of VHF, UHF and microwave signal beyond the horizon (LOS).

→ The troposphere is nearest portion of atmosphere about 15km.



Tropospheric scattering propagation is also called as forward scatter propagation. (8)

→ The scattering propagation depends on two accepts. (i) Ionospheric propagation (ii) out come of scattering layers from troposphere.

3.:- → The Tropospheric scatter propagation occurs due to air-turbulance, irregular and discontinuities in the atmosphere, to divert a small fraction of Radio energy transmitted towards Receiving antenna.

→ Generally the radio waves diffract (or) bend along the curved surface of earth.

→ Due to such disturbances & discontinuities there is a small irregularity in the refractive Index.

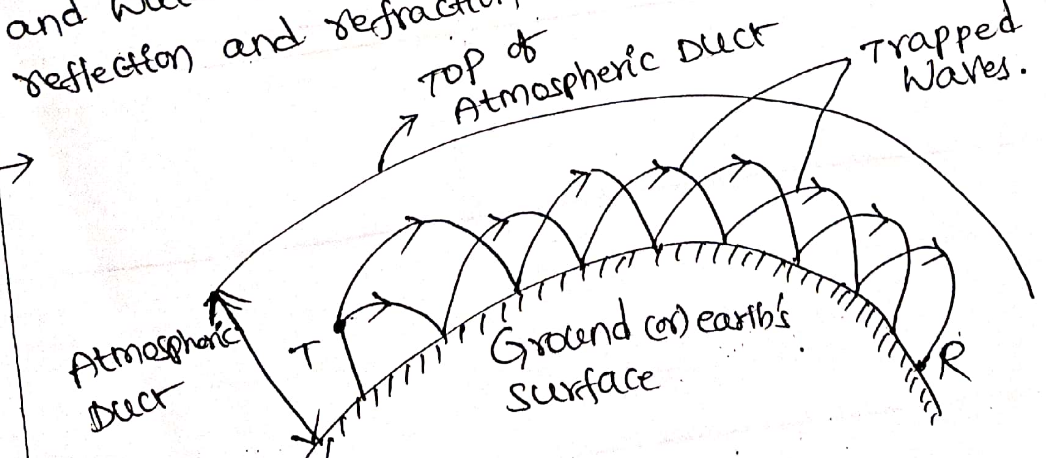
Duct propagation:- [Super Refraction]

→ The Duct is a leaky waveguide through which E.M waves move in the air by successive reflection and refraction. when the signal move through different layers, signal may suffer from some Loss.

→ The VHF, UHF and Micro wave frequencies, which cannot propagate along earth surface and cannot reflect from Ionosphere.

→ In the air region there are different temperature conditions and water vapours states besides these conditions scattering, reflection and refraction combinely called as "Duct propagation".

→



T= Transmitting antenna

...ving Antenna

→ In the Air Region $\frac{d\mu_m}{dh}$ (or) $\frac{dM}{dh}$ is negative. So the height is Increased and μ_m is decreased. If the height is decreased then μ_m is Increased.

→ The energy originating from air region, the electromagnetic Waves are propagating around the curved surfaces.

→ In the troposphere dielectric Constant is greater than unity

→ In the Normal atmosphere (or) standard atmosphere the dielectric constant is decreased with a height value of unity at which the air density is zero.

→ Finally the DUCT effect can be removed by exceeding the maximum wavelength.

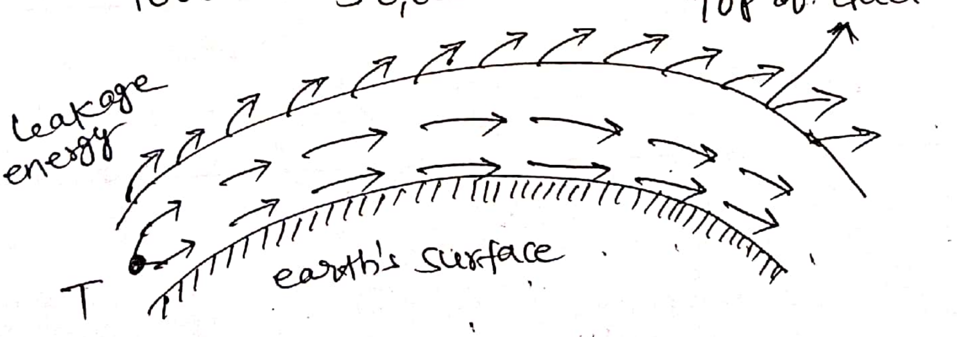
It is given by $\boxed{\lambda_{max} = 2.5\, h_d \sqrt{\Delta \mu_m \times 10^{-6}}}$

$\mu_m = M$ = modified refractive Index.

h_d = height of duct

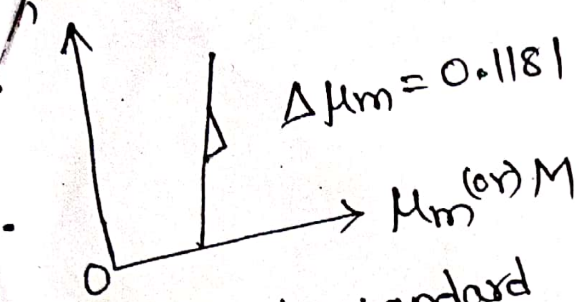
λ_{max} = maximum Wavelength.

λ_{max}	h_d (m)
1	500
10	2300
100	10,700
1000	50,000.

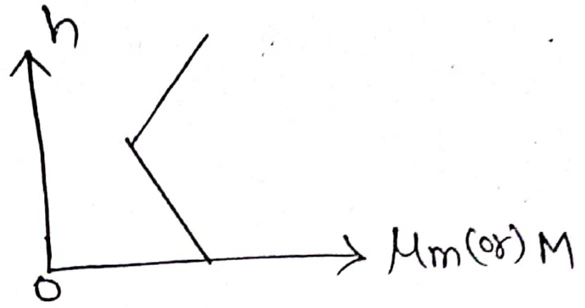


characteristics of M-curves :-

(a) ground standard atmosphere
(b) Refraction at low height
(c) Ground based duct
(d) elevation duct.



(a) ground standard atmosphere.

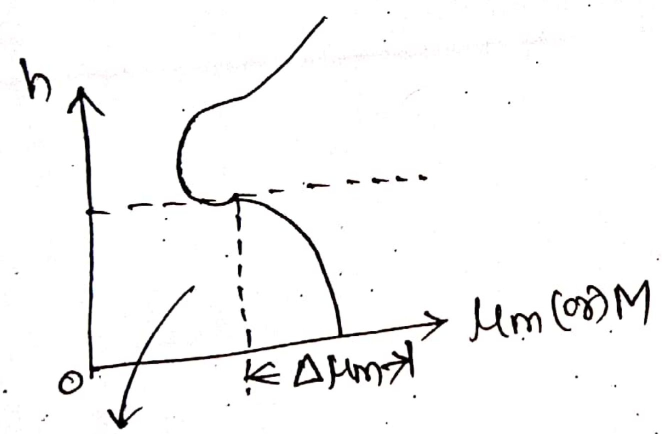


(b) Refraction at low height

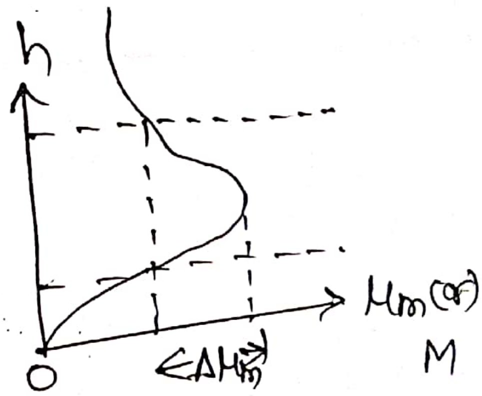
Where

μm = M = modified Refractive Index

h = height of duct

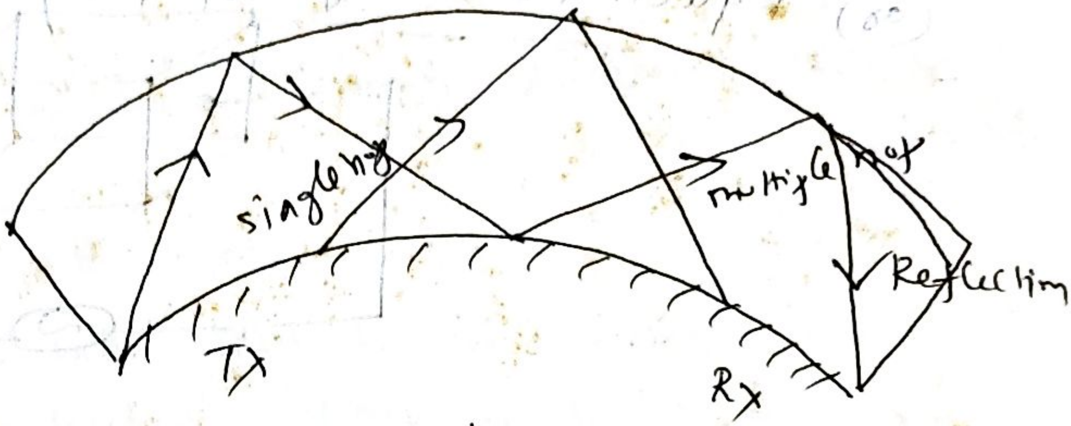


(c) ground Based duct



(d) Elevation Duct

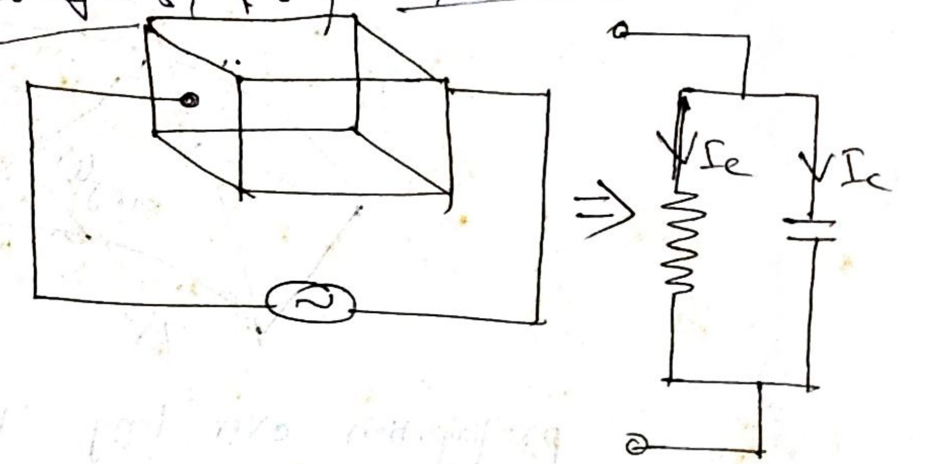
Sky wave propagation :-



propagation over long distance an order of thousand kilometers is not possible by ground wave and space wave propagation.

sky wave are reflected from some of ionization layers of ionosphere and return back to earth in single hop (or) multiple hops. maximum range of communication using single hop is approximately 4000 km. By using the multiple - hops communication we can cover whole world. So, by using ionosphere we can cover any distance around the earth as shown in figure.

propagation of Radio waves through the Ionosphere
(or) Expression for the Refractive index of the Ionosphere.
(or) Mechanism of Reflection & Refraction
(or) Reflection & law of high frequency.



In an ionized medium having free electrons and ions when the radio wave pass through, it set these char-ged particles in motion. The radio waves passes through the ionosphere is influenced by the electrons only and the electrons of ionosphere get motion due to the electric field of radio waves. These electrons vibrate simultaneously parallel to the electric field of the radio wave and these represent a AC current proportional to the velocity of vibration the current will be indu-ctive type. The actual current flowing through a volume of the space in the ionosphere consists usually capaciti-ve current which leads the voltage by 90° and hence electron current subtracted from the capacitive current.

Thus free electron in space decrease the current and so the dielectric constant of the space is also reduced below the value that would be in the absence of electron. So, this reduction causes the path of radio waves to bend toward earth i.e from high electron density to lower density.

Let an electric field of volume

$$E = E_m \sin\omega t \quad V/m$$

is acting across a cubic metre of space in the ionosphere.

Force exerted by electron field on each electron is given by

$$F = -eE \quad (N)$$

Let us assume that there is no collision, then the electron will have velocity v m/sec. in the direction opposite to the field.

$$\text{force} = \text{mass} \times \text{Acceleration}$$

$$-Ee = m\frac{dv}{dt}$$

$$\frac{dv}{dt} = -\frac{eE}{m} \quad (\text{or}) \quad dv = -\frac{eE}{m}\,dt$$

integrating both sides, we get

$$v = \int -\frac{eE_m \sin\omega t}{m}\,dt$$

$$v = \frac{e}{m\omega}E_m \cos\omega t$$

If cubic have the N the number of electrons per cubic metre, then

$$i_e = -Nev \quad A/m^2$$

$$\boxed{I_c = -\left(\frac{Ne^2}{m\omega}\right)E_m \cos\omega t} \quad (A/m^2)$$

which shows that i_e lags behind the electric field by 90°, Beside this Inductive (or) conduction current

displacement current (or) capacitive current (I_c)

$$I_c = \frac{\partial D}{\partial t} = \frac{d}{dt}(K_0E) = K_0\frac{d}{dt}(E_m \sin\omega t)$$

$$D = \varepsilon_0 E = K_0 E$$

$$I_c = \varepsilon_0 \omega E_m \cos\omega t$$

The total current I that flow through a cubic metre of ionized medium is,

$$i = i_c + i_e = K_0 \omega E_m \cos\omega t - \frac{Ne^2}{m\omega} E_m \cos\omega t$$

$$= \omega E_m \cos\omega t \left[K_0 - \frac{Ne^2}{m\omega^2}\right]$$

$$i = \omega E_m \cos\omega t K$$

K = effective Dielectric constant

$$K = K_0 - \frac{Ne^2}{m\omega^2} = K_0\left[1 - \frac{Ne^2}{m\omega^2 K_0}\right]$$

Relative dielectric constant $K_r = \frac{K}{K_0} = 1 - \frac{Ne^2}{m\omega^2 K_0}$

Thus the relative refractive index of the ionosphere w.r.t Vaccum

$$\mu = \sqrt{K_r} = \sqrt{\frac{K}{K_0}} = \sqrt{1 - \frac{Ne^2}{m\omega^2 K_0}}$$

$$v = \frac{c}{\sqrt{K_r}} = \frac{c}{\mu}$$

$m = 9.107 \times 10^{-31}$ kg $\quad e = 1.602 \times 10^{-19}$ C

$K_0 = 8.854 \times 10^{-12}$ F/M $\quad \omega = 2\pi f$

$$\boxed{\mu = \sqrt{1 - \frac{81 N}{f^2}}}$$

Where N – no of electrons per cubic meter (or) Ionic density

f – frequency in Hz

Radio wave bending by the Ionosphere

The bending of radio waves by the ionosphere can be easily understood by the refractive index.

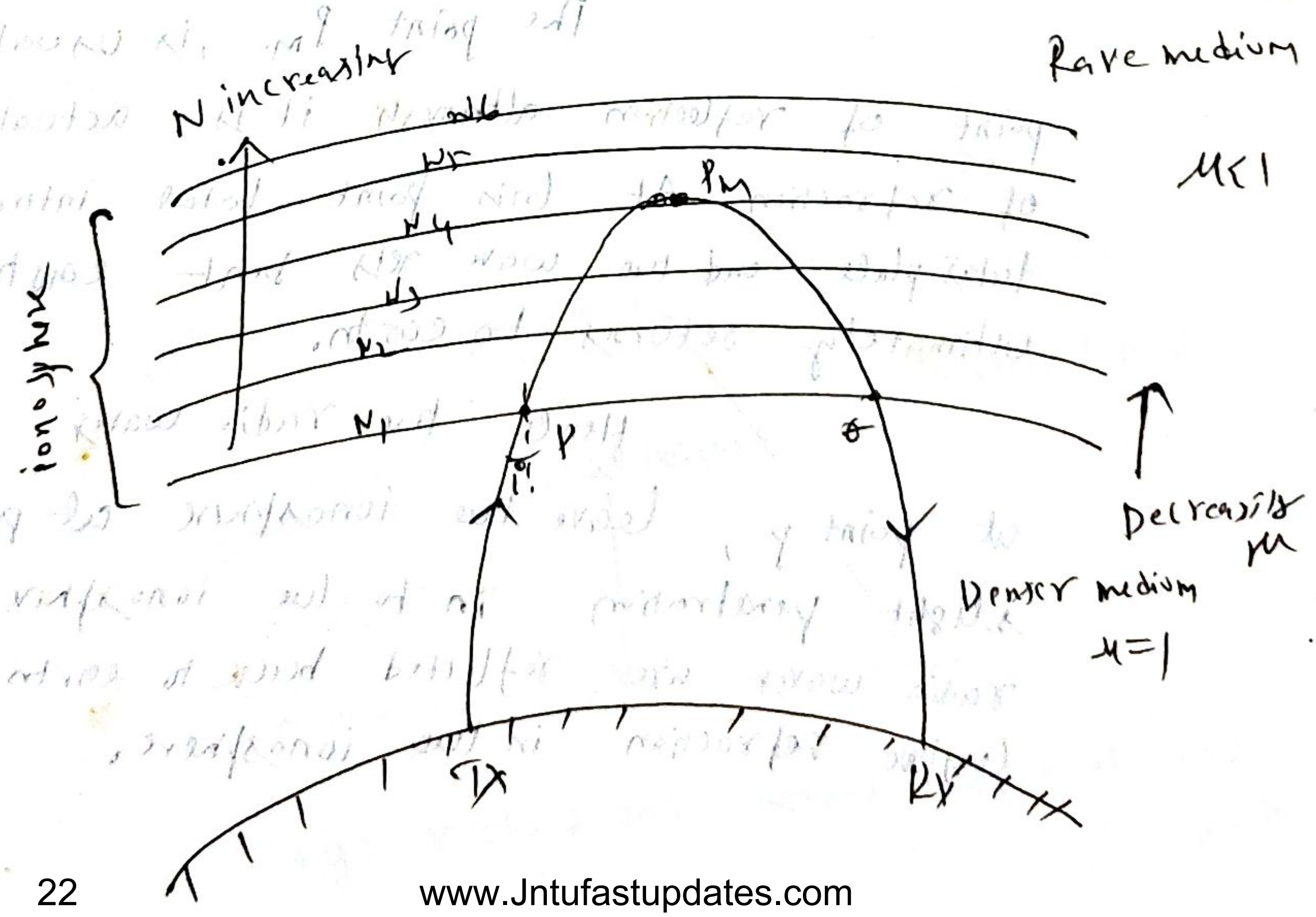
$$\mu = \sqrt{K_r} = \sqrt{1 - \frac{81N}{f^2}}$$

If $f^2 > 81N$, then $\mu < 1$ (real values)

If $f^2 < 81N$, then μ is imaginary which means under such conditions the radio waves are attenuated at this frequency and ionosphere is not able to transmit (or) bend the radio waves.

The bending of radio waves by the ionosphere is governed by Snell's law.

$$\mu = \frac{\sin i}{\sin r} \quad \begin{array}{l} i - \text{incident angle} \\ r - \text{refraction angle.} \end{array}$$



since $\mu < 1$ for the ionosphere, so $\sin i < \sin r$ i.e. angle of refraction will go on deviating from the normal as the wave will encounter rarer medium as shown in fig.

If successive layers of the ionosphere are of higher electron density i.e. $N_6 > N_5 > N_4 > N_3 > N_2 > N_1$ it means μ will go on decreasing and decreasing i.e $\mu_1 > \mu_2 > \mu_3 > \mu_4 > \mu_5 > \mu$ Thus a wave enters at say point p will be deviating more and more and a point will reach where it travels parallel to earth (at P_m). Here the angle of refraction is 90° and the point P_m is the highest point in the ionosphere reached by the radio wave.

$$\boxed{\mu_M = \sin i_m}$$

The point P_m, is usually called as point of reflection although it is actually a point of refraction. At this point total internal reflection takes place and the wave gets bent earthward and ultimately returns to earth.

Hence the radio waves once enter at point p, leave the ionosphere at point Q after slight penetration in to the ionosphere and thus radio waves are reflected back to earth after successive refraction in the ionosphere.

CRITICAL FREQUENCY (f_c):-

The critical frequency of ionized layer of the ionosphere can be defined as the critical frequency in the highest frequency which can be reflected back to earth by a particular layer at vertical incidence. critical frequency is different for different layers, it is denoted by f_c.

$$\mu = \frac{\sin i}{\sin r} = \sqrt{1 - \frac{81N}{f^2}}$$

By definition $i = 0$, $N = N_{max}$ & $f = f_c$

$$\frac{\sin 0}{\sin r} = \sqrt{1 - \frac{81N_{max}}{f_c^2}}$$

$$\boxed{f_c = 9\sqrt{N_{max}}}$$

When f_c - MHz
N_{max} - per cubic metre.

VIRTUAL HEIGHT

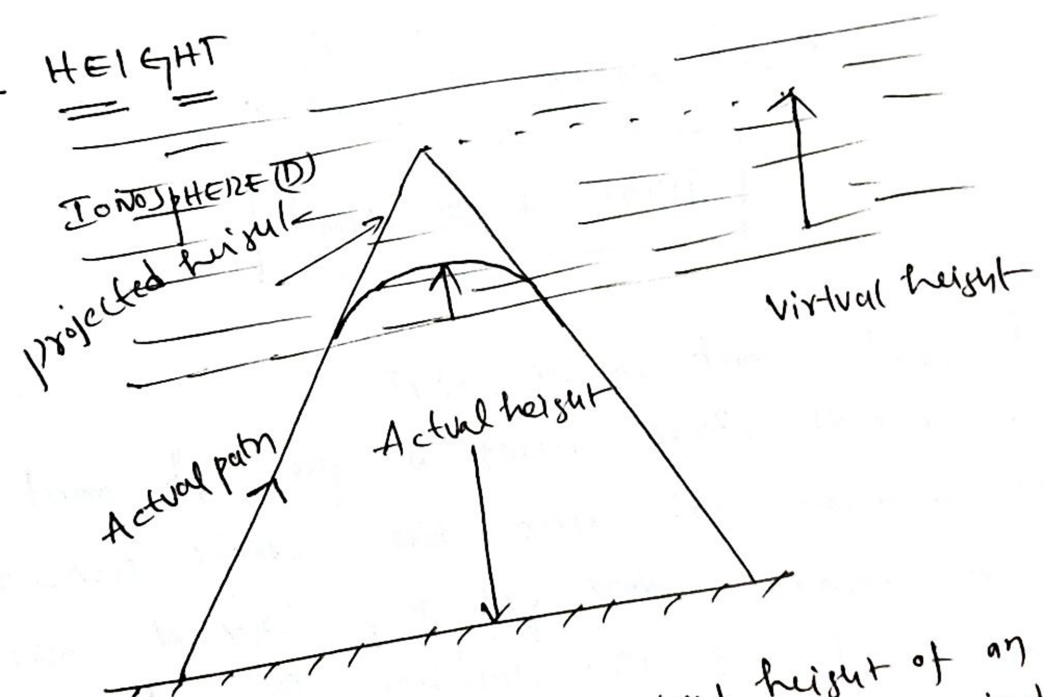


Fig: virtual and actual height of an Ionized layer.

MAXIMUM USABLE FREQUENCY (MUF)

critical frequency is the maximum frequency of the radio wave which is returned from a ionized layer at vertical incidence.

MUF :- ~~It is~~ the maximum possible value of frequency for which reflection takes place for a given distance of propagation, is called as the maximum usable frequency for that distance, and for the given ionosphere layer.

For a sky wave to return to earth, angle of reflection i.e $r = 90^\circ$

$$\mu = \frac{\sin i}{\sin r|_{r=90^\circ}} = \sqrt{1 - \frac{81 N_m}{f_{muf}^2}}$$

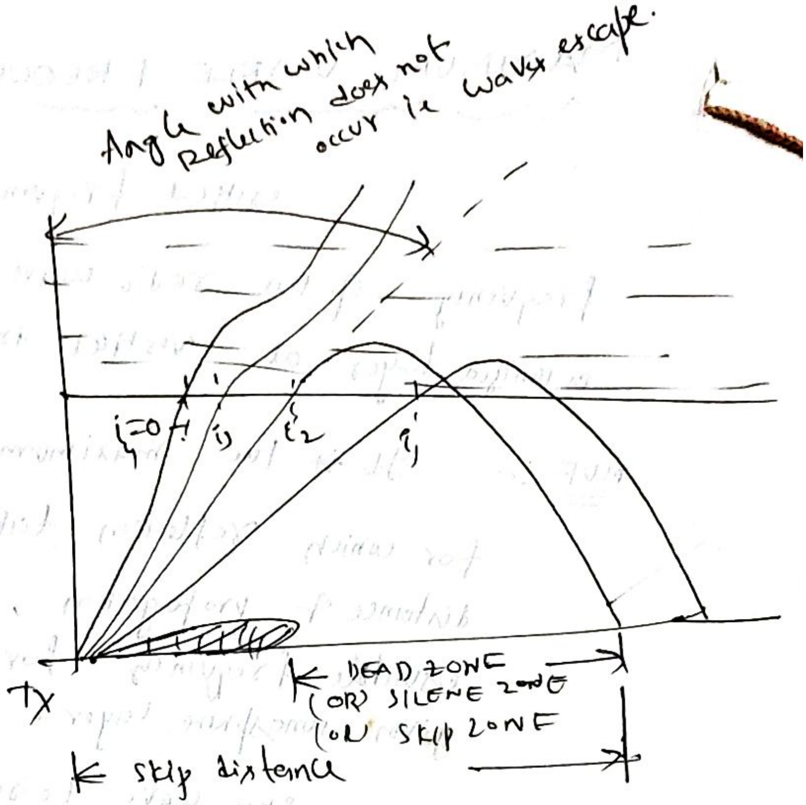
$$1 - \sin^2 i = \frac{81 N_m}{f_{muf}^2}$$

$$f_{muf}^2 = 81 N_m \sec^2 i$$

$$\boxed{f_{muf} \leq f_c \sec(i)}$$

This means that f_{muf} is greater than f_c by a factor $\sec i$. This is known as SECANT LAW and gives the maximum frequency which can be used for sky wave communication for a given angle of incidence (i) b/n two points on the earth.

SKIP DISTANCE



~~The~~ skip distance: The distance at which surface wave becomes negligible and the distance at which the first wave returns to earth from the ionospheric layer, there is a zone which is not covered by any wave. This is skip zone and distance across it is the skip distance.

The skip distance is the shortest distance from a transmitter, measured along the surface of the earth, at which a sky wave of fixed frequency will be returned to earth is known as skip distance.

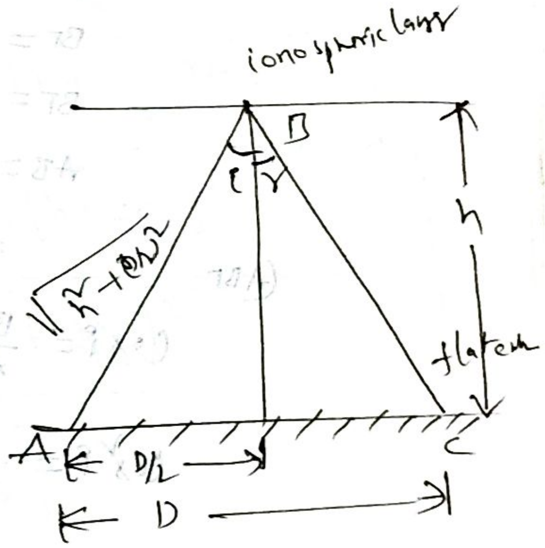
As the angle of incidence at the ionosphere decreases, the distance from the transmitter, at which the ray returns to ground first decreases. This behavior continues until eventually an angle of incidence is reached at which the distance becomes minimum. The minimum distance is called skip distance 'D'.

with further decrease in angle of incidence, the wave penetrates the layer and does not return to earth, infact, skip distance is the distance skipped over by the sky wave.

CALCULATION OF MUF & SKIP DISTANCE

Case①: when earth is flat:-

The ionozed layer may be assumed to be thin layer with sharp ionization density gradient, which gives mirror like reflection of radio waves as shown in fig. For shorter distance the earth can be assumed to be flat.



From $\angle OAB$

$$\cos i = \frac{BO}{AB} = \frac{h}{\sqrt{h^2+(D/2)^2}}$$

The MUF for which the wave is to be reflected from the layer for returning to earth

$$\mu = \sin i = \sqrt{1 - \frac{81N_m}{f_{muf}^2}}$$

$$\cos^2 i = \frac{f_c^2}{f_{muf}^2}$$

h – height of layer

D – propagation distance (skip)

$$\frac{f_{muf}^2}{f_c^2} = \frac{4h^2+D^2}{4h^2}$$

$$\boxed{f_{muf} = f_c\sqrt{1+\left(\frac{D}{2h}\right)^2}} \text{ (Hz)}$$

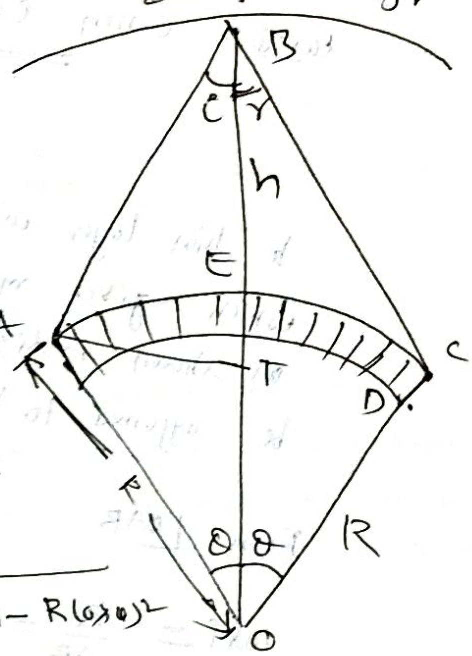
The skip distance

$$\left(\frac{D}{2h}\right)^2 = \frac{f_{muf}^2}{f_c^2} - 1$$

$$D = 2h\sqrt{\left(\frac{f_{muf}}{f_c}\right)^2 - 1} \text{ (m)}$$

Case ②: when earth is curved :-

It earth is curved, then reflecting region is consider to be concentric with earth as shown in the figure. In this figure transmitting wave leaves the transmitter tangentially to the earth. Let 2θ be the angle substanded by the transmission distance 'D' at the centre of the earth O then



Arc = Angle x radius

$$\boxed{D = 2\theta \times R}$$

$\triangle OAT$: $AT = R\sin\theta$

$OT = R\cos\theta$

$BT = OE + EB - OT$

$BT = R + h - R\cos\theta$

$AB = \sqrt{AT^2 + BT^2}$

$= \sqrt{(R\sin\theta)^2 + (R+h-R\cos\theta)^2}$

$\triangle ABT$

$$\cos i = \frac{BT}{AB} = \frac{h+R-R\cos\theta}{\sqrt{(R\sin\theta)^2+(h+R-R\cos\theta)^2}}$$

$$\cos^2 i = \frac{f_c^2}{f_{muf}^2} = \frac{h+R-R\cos\theta}{(R\sin\theta)^2+(h+R-R\cos\theta)^2}$$

The curvature of earth limits both MUF and skip distance D and the limit is obtain when waves leave the transmitter at a grazing angle ($\angle OAB = 90°$)

Thus when D is maximum, θ is maximum

$$\cos\theta = \frac{OA}{OB} = \frac{R}{R+h}$$

However actual value of 'θ' is very small,

$$\cos\theta = \frac{R}{R(1+\frac{h}{R})} = (1+\frac{h}{R})^{-1}$$

$$\cos\theta = 1 - \frac{h}{R} \qquad \text{Because } \frac{h}{R} << 1$$

$$1 - 2\sin^2\frac{\theta}{2} = 1 - \frac{h}{R}$$

$$1-2\left(\frac{\theta}{2}\right)^2 = 1-\frac{h}{R}$$

$$1-\frac{\theta^2}{2} = 1-\frac{h}{R}$$

$$\theta^2 = 2\frac{h}{R}$$

$$D^2 = 4R^2\theta^2 = 4R^2 \times \frac{2h}{R} = 8Rh$$

$$h = \frac{D^2}{8R}$$

$$\cos\theta = 1-\frac{D^2}{8R^2}$$ and as θ is small

$$\sin\theta = \theta = \frac{D}{2R}$$

$$\frac{f_c^2}{(f_{muf})^2_{max}} = \frac{h+R-R\left(1-\frac{D^2}{8R}\right)}{\left\{R^2 \cdot \frac{D^2}{4R^2} + \left[h+R-R\left[1-\frac{D^2}{8R}\right]\right]\right\}^2}$$

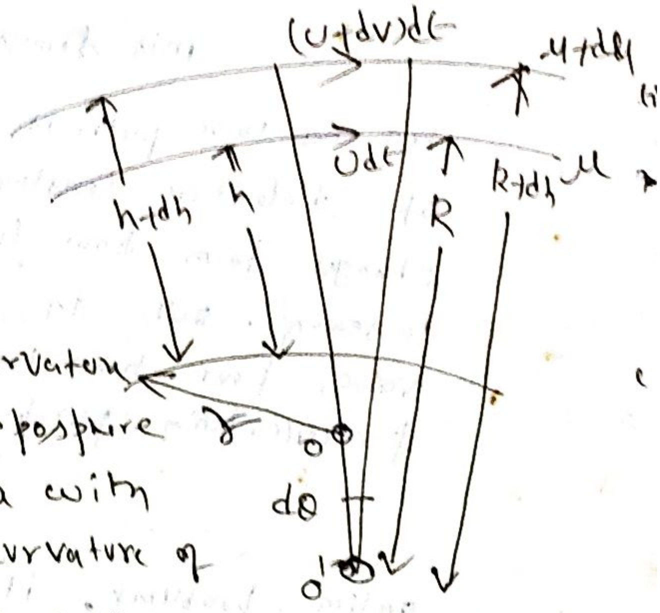
$$= \frac{\left[h+\frac{D^2}{8R}\right]^2}{\frac{D^2}{4}+\left[h+\frac{D^2}{8R}\right]^2}$$

$$\boxed{(f_{muf})_{max} = \frac{\sqrt{\frac{D^2}{4}+\left[h+\frac{D^2}{8R}\right]^2}}{\sqrt{\left[h+\frac{D^2}{8R}\right]^2}}}$$

$$\boxed{(D_{skip})_{max} = \sqrt{8hR}}$$

$$\boxed{D = 2\left(h+\frac{D^2}{8R}\right)\sqrt{\left(\frac{f_{muf}}{f_c}\right)^2_{max} - 1}}$$

Effective Earth's Radius



Refraction of Radio waves in the troposphere.

Let us now derive a relation b/n the radius of curvature of the ray path in the troposphere & and change of refractive index with height by assuming the curvature of the earth. Consider a radio wave which is travelling nearly horizontally in the troposphere and its path is bent into an arc by the variation of the refractive index with height as shown in fig.

v — velocity of propagation

h — Height above the earth

R — Radius of curvature of the ray path

r — Actual radius of earth.

$$d\theta = \frac{v\,dt}{R}$$ radius × angle = arc

$$R\,d\theta = v\,dt$$

$$(R+dh)\,d\theta = (v+dv)\,dt$$

$$dh\,d\theta = dv\,dt \quad \text{(or)} \quad \frac{d\theta}{dt} = \frac{dv}{dh}$$

$$v = \frac{c}{\sqrt{\epsilon_r}} = \frac{c}{\mu}$$

μ — refractive index at height h,

$$\frac{dv}{dh} = -\frac{c}{\mu^2}\frac{d\mu}{dh} = -\frac{v}{\mu}\frac{d\mu}{dh}$$

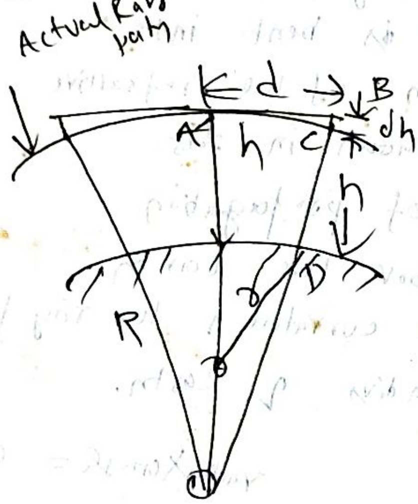
$$\boxed{\frac{dv}{dh} \simeq -v\frac{d\mu}{dh}} \quad (\because \mu \simeq 1)$$

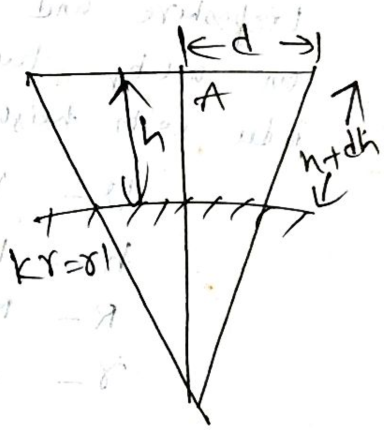
$$R = \frac{v\cdot dt}{d\theta} = \frac{v}{\frac{d\theta}{dt}} = \frac{v}{-v\frac{d\mu}{dh}}$$

$$\boxed{R = -\frac{dh}{d\mu}}$$

This shows that radius of curvature of the wave path is a function of the rate of change of dielectric constant (or) refractive index with height, changes from hour to hour, day to day and season to season. But in practice, however, an average value, four times the radius of the earth is used for calculation purposes.

In actual working with propagation problems, it is usually convenient to represent ray path as straight lines instead of being curved.





$$d^2 + (r'+h)^2 = \{r'^2 + (h+dh)\}^2$$

$$d^2 = 2\delta h\,(r'+h) \quad \because \delta h^2 << 2\delta h\,(r'+h)$$

$$dh = d^2/2r'$$

$$DC = d^2/2R \qquad BD = \frac{d^2}{2r}$$

$$k = \frac{1}{1-\frac{r}{R}}$$

$$BC = \delta h = BD - DC$$

$$dh = \frac{d^2}{2r} - d^2/2R$$

$$r' = \frac{r}{1-\frac{r}{R}}$$

$$\frac{d^2}{2r'} = \frac{d^2}{2r} - \frac{d^2}{2R}$$

$$k = \frac{1}{1+r\frac{de}{dh}}$$

$$\frac{1}{r'} = \frac{1}{kr} = \frac{1}{r} - \frac{1}{R}$$

$$\frac{de}{dh} = 0.046\times10^{-6}\ \text{per}/\text{mt}$$

$$k = r'/r$$

if radius of curvature R of ray path is equal to 4 times the actual earth's radius, then effective radius of earth is $\frac{4}{3}$ times the actual radius of earth

$$\frac{dn}{dh} = 0.040\times10^{-6} \text{ per/metre}$$ for standard atmosphere

& $r = 6370km$.

$$k = \frac{1}{1-6.37\times10^{6}\times0.040\times10^{-6}}$$

$$k = \frac{4}{3} \qquad \Rightarrow k = \frac{r'}{r} = \frac{4}{3}.$$

Hence for a standard atmospheric refraction the effective earth's radius is $\frac{4}{3}$ times the actual earth's radius.

$$d = \sqrt{2r}\,[\sqrt{h_t}+\sqrt{h_r}]$$

So the modified Los

$$d = \sqrt{2r'}\,[\sqrt{h_t}+\sqrt{h_r}]$$

$$= \sqrt{2\times\frac{4}{3}\,6370\times10^{3}}\,(\sqrt{h_t}+\sqrt{h_r})$$

$$\boxed{d = 4.12\,[\sqrt{h_t}+\sqrt{h_r}]\ \text{KM}}.$$

This is the eqn for calculating radio horizon (or) line of sight distance. when ht & hr given in meters.

$$\boxed{d = 1.414\,[\sqrt{h_t}+\sqrt{h_t}]\ \text{miles}}$$

when ht & hr in feets.

Effect of Earth's curvature on Tropospheric propagation

on the tropospheric propagation

the following two effects are introduced by the curvature of the earth.

(i) The difference in path lengths b/n direct and ground reflected waves is reduced as the point of reflection on the ground is raised. As a result it tends to reduce the signal strength at receiving point.

(ii) Further, since the reflection at the ground takes place at a spherical point rather than a flat point and hence the reflected ray becomes divergent which results in weaker at receiving point. this tends to increase the field strength at the receiving points of the total space wave